ARPA ORDER NO. 189-1

MEMORANDUM
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DECEMBER 1968

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MEASUREMENT OF TECHNICAL PERFORMANCE IN WEAPON SYSTEM DEVELOPMENT PROGRAMS: A SUBJECTIVE PROBABILITY APPROACH

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PREPARED FOR
ADVANCE RESEARCH PROJECTS AGENCY

The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum is a product of a continuing study of advanced techniques for the management of research and development. It presents an exploratory effort to develop the framework of a procedure for the collection and analysis of data on uncertainty and progress regarding technical performance in weapon system development. The data collected are concerned with uncertainty about the characteristics of the component parts of the total system. These data are combined, using system design relationships, to determine the uncertainty about the performance of the total system. Application of the procedure will involve the use of Monte Carlo simulation.

The procedure may be applied to military or commercial development projects, and it may be used to obtain information on uncertainty at the system, subsystem, or other levels. As such, it should be of interest to persons involved in the management of research and development.

SUMMARY

The purpose of this Memorandum is to present a conceptual framework for measuring uncertainty and progress regarding technical performance in a weapon system development project. The emphasis is on information of interest to higher-level military and contractor management. For this reason, the discussion is in terms of characteristics such as would appear in an operational requirement describing the performance of the system.

The procedure is based on four premises. First, progress in systems development is characterized by reduction of uncertainty; hence, if uncertainty can be measured at different times, then progress may be indicated by changes in measures of uncertainty. Second, assessment of uncertainty regarding systems development is subjective; that is, it depends on individual beliefs. Third, subjective estimates of uncertainty can be expressed in terms of probabilities. Fourth, the amount of uncertainty is indicated by statistical measures of appropriate probability distributions.

Consistent with these premises, the procedure is concerned with obtaining the probability distributions for the critical system performance characteristics. The following steps are involved:

1. Find design equations that relate subsystem properties to total system performance.
2. Obtain subjective probabilities for subsystem and component properties that influence total system performance.
3. Use Monte Carlo procedures to generate probability distributions for the system performance characteristics.
4. Calculate statistical measures of system performance probability distributions.
5. Compare statistical measures for different time periods to obtain indications of progress.

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I. INTRODUCTION

Meaningful information on the status and progress of a weapon system during its development is a critical input to enlightened military decisionmaking. Major decisions regarding program acceleration, continuation at the present rate, change in direction, or possible cancellation depend heavily on careful evaluation of such information. However, several years are generally required before the product of a weapon development program can be subjected to total system testing and the overall performance definitively evaluated. Hence, decisionmaking in the interim must be accomplished with imperfect information--imperfect in the sense that the outcome is uncertain. The best decisionmaking in these circumstances must take explicit account of the uncertainty. This can be done by obtaining estimates of the various possible values of the future performance of the weapon system. This Memorandum describes the framework of a procedure that takes explicit account of uncertainty, that generates estimates of the various possible values of future performance, and that uses these estimates to assess status and progress.*

The major characteristics underlying the approach are the following:

1. The development of a new, operational military system involves combining a number of subsystem and component technologies--some new or improved and some current--to arrive at a new system capability. At the beginning of these projects two types of information are available: design equations describing performance of the general class of system in terms of the characteristics of the components, and some knowledge about new or improved technologies to be incorporated in certain of the components or subsystems. On the other hand, it is not known whether these technologies can be engineered and combined into a system configuration that will deliver the desired new capability.

*The relationship of this procedure to the overall process of development is illustrated in Appendix A.

2. Status in development can be indicated by a measure of uncertainty, and progress by change in status.* Hence, progress is indicated by reduction of uncertainty. Weapon system development has been described as the sequential purchase of information regarding some object about which the state of knowledge is incomplete.** The technical status of a development project, at any time, is best indicated by the accumulated knowledge at that time. Progress is measured by periodic assessment of the change in the state of knowledge--that is, the information obtained--since the last review.

3. The appropriate variables for consideration in decisionmaking at the total system level can be expressed in a few critical system performance characteristics. The significant characteristics are those that indicate the capability of the system to perform certain prescribed missions. For example, in the case of an aircraft, the characteristics might be speed, range, altitude, etc.***

4. To obtain such measures for the performance of the total weapon system requires a method for systematically analyzing the implications of knowledge gained at the subsystem and component levels. Therefore, the appropriate variables for data collection are the physical and performance characteristics of major subsystems and components. This is so because, in general, the most technically knowledgeable people involved in a system development project are the engineers doing the work. There is no person who can appraise the performance of the system with anything near the same level of confidence that an engineer can appraise the performance of the portion of the system upon which he is working.

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*Measurement of status in development has a different basis than measurement of status in production. In production, status is measured in terms of how many parts are manufactured, assembled, etc. The issue of primary concern is when the total job will be done. In development, the main concern is how the end product will perform. Completion of development is indicated by the fact that the part's characteristics are known with certainty.

**See Ref. 17, pp. 1-3.

***These characteristics are usually stated in a Required Operational Capability (ROC) or a Requirements Action Directive (RAD).

5. Because weapons development is not a random process, the use of <u>subjective probability distributions</u>* and quantities derivable from them is necessary in measuring the technological uncertainty** involved.

The uncertainty and progress assessment technique described in this Memorandum is based on the above characteristics, and it involves the following steps:

1. Determine design equations that relate subsystem properties to total system performance.
2. Obtain subjective probabilities for subsystem and component properties that influence total system performance.
3. Use Monte Carlo procedures to generate probability distributions for the system performance characteristics.
4. Calculate statistical measures of system performance probability distributions.
5. Compare statistical measures for different time periods to obtain an indication of progress.

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*A subjective measure must be used because it is impossible to obtain an objective measure. This is a consequence of the methodological differences between <u>measuring</u> something that has occurred as opposed to <u>predicting</u> something that is going to occur. If a measuring device exists that can be applied to an event that has occurred, then it can be said that the measurement is <u>objective</u>, and there is certainty regarding what has happened. In the case of predicting which one of several events will occur, the situation is personal, or <u>subjective</u>, but the likelihood can be expressed quantitatively in terms of probabilities. These probabilities, along with certain summary statistics, indicate the degree of uncertainty. In some instances, probabilistic evaluations of the likelihoods of possible future events are referred to as being objective. Probabilities are objective when the events are the results of "random phenomena."

The nature of subjective probabilities and techniques for obtaining them are described in Appendix B. Some of the statistical measures that characterize probability distributions are reviewed in Appendix D. For further information on random phenomena, see Ref. 24, p. 2.

**This type of uncertainty can be distinguished from several other types; for example, the uncertainty regarding the enemy and his reactions. (See Ref. 14, pp. 15-16) In general, the other types of uncertainty are external to the weapons developer (contractor) and therefore not directly relevant to the evaluation of contractor performance.

Note that the measures of status and progress are both related to the state of knowledge regarding the end product and both are based on the same subjective inputs and design equations. The use of design equations and carefully determined subjective probabilities will assist in obtaining measures of uncertainty and progress in which caprice and unreasoned intuition are held to a minimum.

II. MEASUREMENT OF TECHNOLOGICAL UNCERTAINTY (STATUS)

This section deals with the various levels and times at which subjective probabilities may be obtained during a development project, and describes a process to generate uncertainty information about the performance of the total system, based on similar information regarding the subsystems and components.

STRUCTURAL ELEMENTS RELEVANT TO MEASUREMENT

The development of a complex system is accomplished by division of effort. As work progresses, this division becomes more and more detailed. Hence, the framework or structure of a weapon system development project can be viewed as consisting of two major elements, namely, (1) decomposition of work into components and (2) the time sequencing of activities.

Decomposition into Work Elements

The division of the total system into its subsystems, the subsystems into their components, and so on is frequently referred to as the work breakdown structure. Figure 1 shows an example of the first four levels in a work breakdown structure for an aircraft development project. In addition, the associated levels of management and their areas of responsibility (performance characteristics) are shown. For example, the major contractor is responsible for the performance of the weapon system--here an aircraft--and therefore for the system performance characteristics.*

The level of weapon system performance may not be the best level at which to obtain subjective probabilities. It seems logical that the "quality" of probability estimates regarding the possible outcome

*Not shown but equally important is a multi-level structure of design equations in which the performance variables associated with subsystems and components are related to the performance objectives for the overall system. This feature will be referred to again.

of activities should be better at low levels of the management structure than at high levels. This is because at high levels a large amount of time is spent on managerial tasks and a small amount on technical tasks. Managerial decisions and actions, although perhaps broader in perspective, frequently have only indirect influence on technical

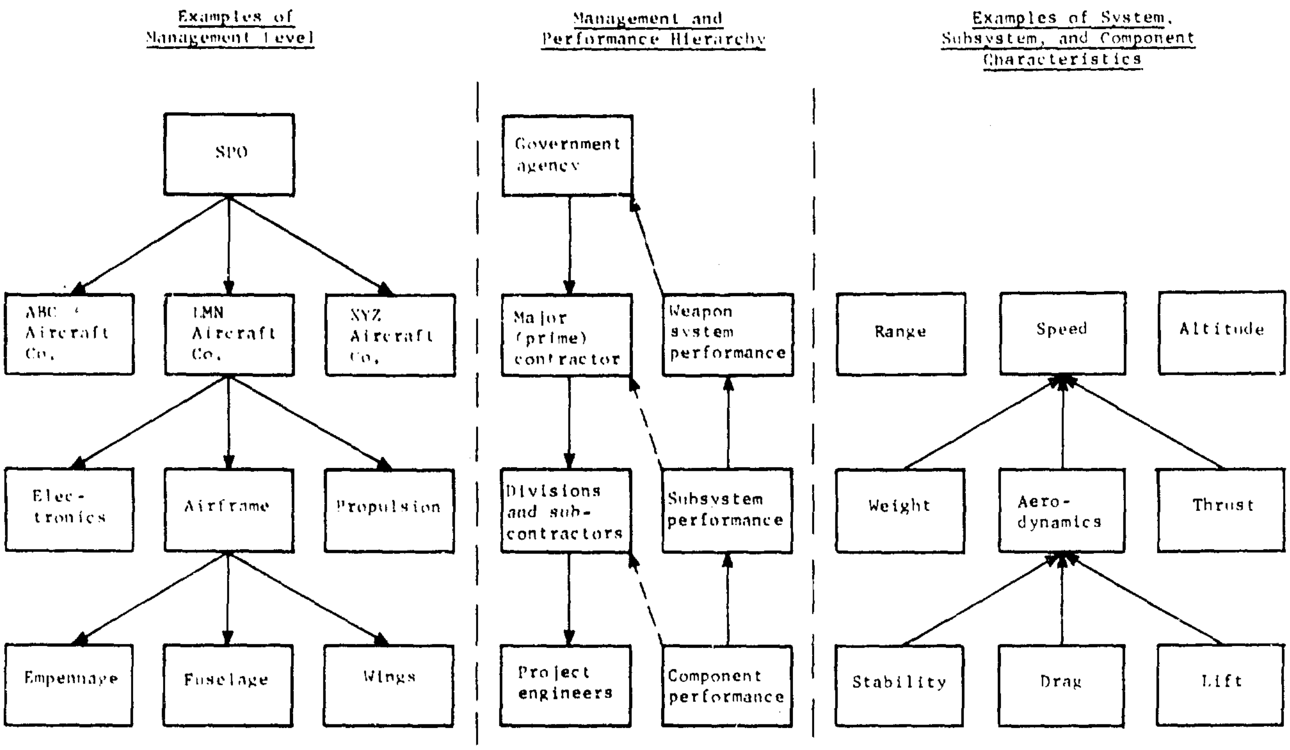

Fig. 1 - An Example of the Management and Performance Levels for an Aircraft Development Project

performance. Detailed knowledge concerning work in progress may be limited. At lower levels knowledge of the individual elements of work is likely to be more intimate, and the decisions and actions more directly related to the characteristics of the components and subsystems. The most detailed knowledge is found at the le· ɔl of the engineers

engaged in design work. Thus, considering the quality of the input data, the "best" approach is to obtain subjective probabilities at the engineering level and use an aggregating process to generate probabilities at the weapon system performance level.

Time Sequencing of Activities

The time sequence of activities is important because it relates to the level of detail to which the work breakdown structure can be defined. At any time before the completion of a project, it generally is difficult, if not impossible, to specify explicitly all of the remaining activities and the order in which they will be undertaken. Figure 2 illustrates a general sequencing of activities for a jet engine development project.

Specification of the time sequence of activities is not a critical element of this study. The important point is that work progresses to finer levels of detail during the course of the project. This is illustrated in Fig. 3. The development of a new operational weapon system begins with an evaluation of needs and opportunities that leads to delineation of desired characteristics for the total system. Prior to the proposal phase, the subsystems and components usually are not analyzed in detail. During the proposal phase, the gross characteristics and key details of the components are analyzed. After the contract is granted, the analysis proceeds down to the level of individual components and parts. By that time, sufficient engineering analyses will have been made regarding state-of-the-art advances and off-the-shelf items to justify obtaining subjective probabilities at these detailed levels. The measurement scheme described in this Memorandum is designed to monitor changes in uncertainty regarding the system's performance, based on information regarding the components and subsystems.

AGGREGATION TECHNIQUES

Having argued that "good" probabilistic estimates can be obtained for the characteristics of the components of a total weapon system that is being developed, it is necessary to demonstrate how these estimates

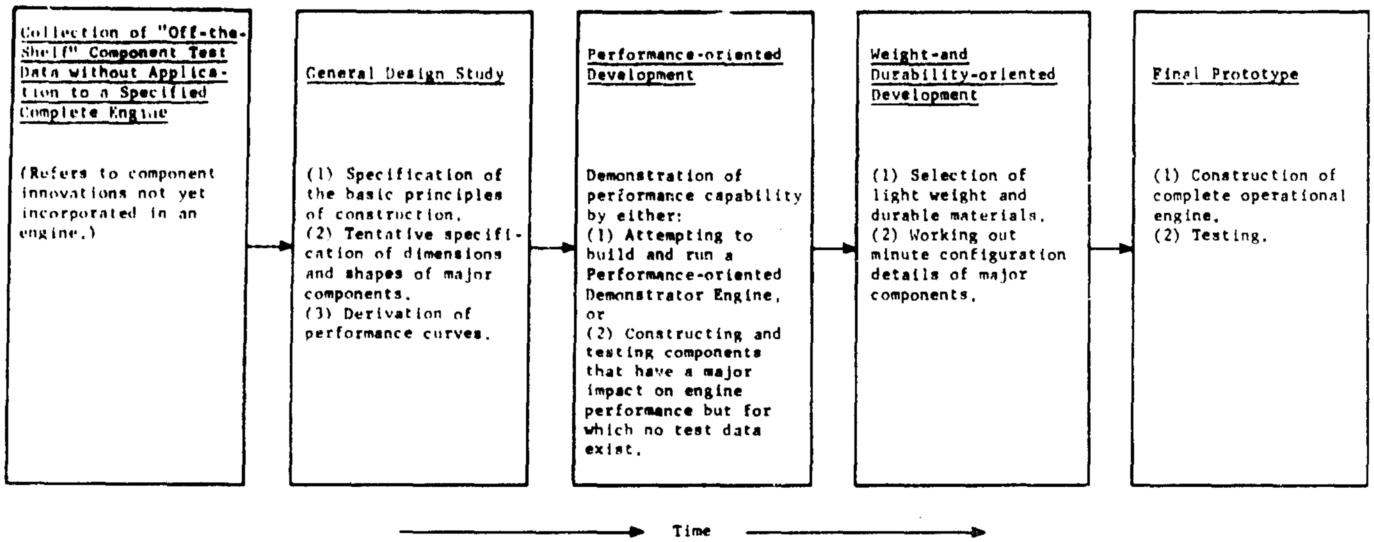

Fig. 2 - A Generalized Sequence of Activities in Jet Engine Development (Adapted from Ref. 22)

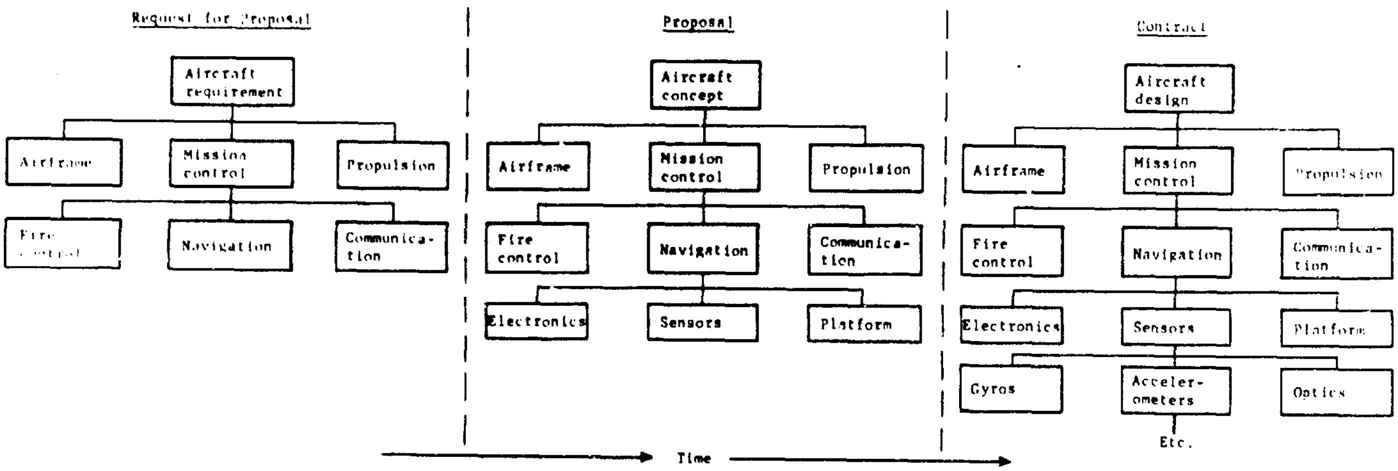

Fig. 3 - An Illustration of Increasing Detail of Design

can be analyzed and manipulated to arrive at information about the performance of the total system. As was referenced above, the performance of a weapon system can be expressed in terms of the characteristics of its components by means of design equations. Some examples are given in the next section.

Given the design equations and probability distributions for the component characteristics that appear in the design equations, then it is possible to generate probability distributions for the system performance characteristics. This can be done analytically; however, mathematical analysis is feasible only when the probability distribution are simple or of standard form and the design equations satisfy certain restrictions.* Because these conditions will almost never be satisfied, analytical techniques appear inappropriate. Instead, simulation must be used.

The method described here uses Monte Carlo simulation, in which the subjective probability distributions for the component characteristics are subjected to random sampling. The values obtained are then used to calculate values for the system performance characteristics. This process is repeated a large number of times until distributions over the possible values of the system performance characteristics are obtained.** In this way, Monte Carlo simulation is used to generate probability distributions for the critical system performance characteristics (see Fig. 4).

The following example demonstrates the nature of the design equations, the probability distributions for the component characteristics, and the probability distributions for the system performance characteristics. Discussion of techniques to explicitly account for various types of interdependence follows the example.

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*For a discussion of the use of analytical techniques with normal probability distributions see Ref. 1. Of course, non-normal distributions can be modified, or the engineers can be questioned in such a way that their responses are "normal." However, this is believed to introduce unnecessary "inaccuracy." See Appendix B.

**Among the numerous ways to determine the exact number of times that the process should be repeated are: (1) to assume that the resultant distributions are normal and use the appropriate statistical rules, and (2) to obtain distributions of large numbers of small samples and thereby take advantage of the law of large numbers. These two courses of action can be evaluated using statistical techniques found in standard texts such as Ref. 23.

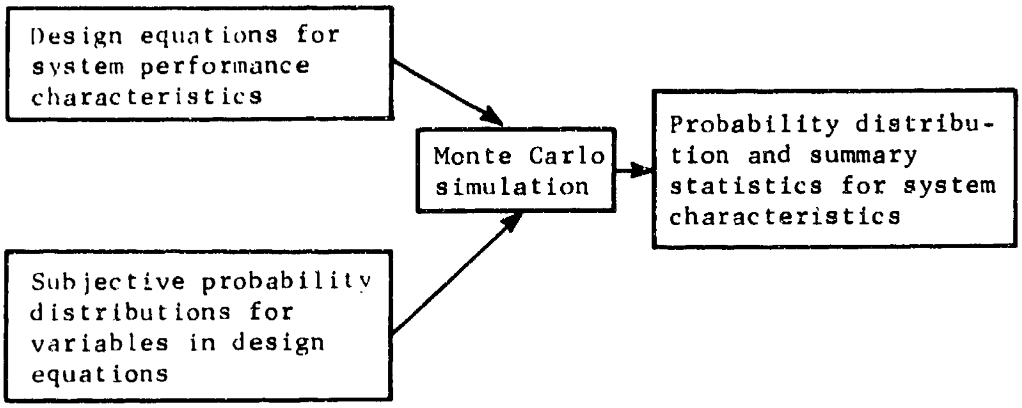

Fig. 4 - The Inputs and Outputs of the Aggregation Process

AN EXAMPLE

Step 1. Obtain Design Equations

A design equation is a functional relationship between a system performance characteristic and the subsystem and component characteristic that determine the system's performance. Such equations are derived from empirical data and they are approximate representations over the total range of the data. Consequently, in many cases more realistic results can be obtained by using the empirical data. The discussion that follows uses design equations exclusively. However, it should be noted that the technique works regardless of the form of the data. The design equations are merely a representation of the data that allows the user to calculate values of the system performances for any combination of component characteristics. A similar result could be obtained by reading the empirical data in tabular or graphical form. In a real application, both design equations and graphical (or tabular) representations would probably be used.

Because of the approximate nature of design equations and the desire of engineers and others to avoid certain types of inadequate design, many of the equations have been modified to include safety, or "fudge" factors. For example, a constant may be added, or subtracted, to provide a built-in margin of safety. These modified equations give a less accurate representation of reality than their unmodified

forms; hence, they are not as appropriate for predicting the characteristics of the total system based on the characteristics of the parts.

The design equations used in the measurement technique should be the equations used by the engineers who are doing the design work. However, when the equations are used for predicting system performance, any known "fudge" factors should be removed.*

The amount of detail in the design equations will depend on how much the various states of the art are being advanced. For example, if an existing engine is to be used, then only the gross characteristics of the engine that influence system performance (for example, thrust, weight, fuel consumption, etc.) need be considered. If a new engine is being developed, then further details of the engine must be considered in order to determine the gross characteristics; that is, a design equation for the engine is necessary.** Thus, there is a hierarchy of design equations, and the level of detail required for assessing status depends on the degree of technological advancement being incorporated.*** The design equations in this summary example are for

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*If there are two or more design equations and there is no agreement about which one is correct, then it may be useful to make a probabilistic combination of the competing design equations.

**There may be cases in which a choice has not been made between using an existing component and developing a new one. Methods for dealing with such cases are discussed at the end of this section.

***At this point it may be useful to summarize the various observations regarding <u>level of detail</u>. It does not seem likely that hard and fast rules will evolve; rather the objective can be stated generally as the minimum detail consistent with obtaining the ability to realistically sense (measure) status. As mentioned above, the technique should be applied in more detail--more levels of design equations--on subsystems and components involving new technology; that is, where advances in state of the art introduce greater uncertainty. Second, the technique should be carried to the level where knowledgeable engineers are most closely involved with those technical aspects of the design which bear directly on ultimate performance of the weapon system. Third, as will be discussed subsequently, it may be desirable to apply the process to an additional level(s) to achieve statistical independence in the basic subjective probability inputs. These considerations must be balanced against the objective of avoiding a technique that is excessively burdensome to either the engineers providing the estimates or the administrative staff processing them and preparing management reports.

illustration only.

Consider a project to develop an aircraft and suppose that the performance characteristics that are critical to the aircraft's mission capability are speed, altitude, range, and endurance. The design equations determine the component characteristics for which subjective probabilities are required.

Speed. The design equation for the maximum, constant altitude, level-flight speed of an aircraft is given by

$$V_{max} = \left\{ \frac{195.5 \left[T_{max} + \left(T^2_{max} - 1.274 \frac{C_{D_o} S W_g^2}{\sigma e b^2} \right)^{\frac{1}{2}} \right]}{S C_{D_o}} \right\}^{\frac{1}{2}}, \qquad (1)$$

where T_{max} is the maximum thrust available (assumed independent of speed), $C_{D(o)}$ is the drag coefficient for zero lift, S is the wing area, W_g is the gross weight of the aircraft, σ is the altitude density ratio,* e is the efficiency factor, and b is the wing span.**

Altitude. One way to determine an aircraft's altitude capability, H_{max}, is to determine the ambient pressure ratio at the absolute ceiling and then use a table of the ambient pressure ratio as a function of altitude to determine the altitude. One form of the equation for the ambient pressure ratio at the absolute ceiling is***

$$P_{ac} = \left(\frac{C_{D_o}}{\pi A e} \right)^{\frac{1}{2}} \frac{(W_p/T)}{\alpha(W_p/W_g)} . \qquad (2)$$

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*Obtained from Standard Atmosphere Table, Ref. 25, pp. 481-482.

**Obtained by solving Eq. (4-2), Ref. 25, for V. This formulation assumes a parabolic variation of C_D versus C_L and does not include consideration of compressibility speed corrections. It is used when the speed is limited by the thrust available.

***Ref. 4, p. 1. Again, this equation is for the thrust-limited ceiling. If the service ceiling is the important characteristic, then the design equation for that quantity should be used.

Using $A = b^2/S$, this can be written as

$$P_{ac} = \left(\frac{SC_{D_o}}{\pi e}\right)^{\frac{1}{2}} \frac{W_g}{\sigma bT} \qquad (3)$$

where S, T, W_g, b, and e are defined above; W_p is the weight of the power plant; and α is the thrust lapse rate factor.

Range. The maximum range (in miles) of a turbojet aircraft at a constant altitude is given by the following equation:*

$$R_{max} = \frac{2}{C'}\left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)_{max}\left(\frac{391W_g}{\alpha S}\right)^{\frac{1}{2}}\left[1 - \left(\frac{W_f}{W_g}\right)^{\frac{1}{2}}\right] . \qquad (4)$$

For parabolic polar variation of C_D versus C_L,

$$\left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)_{max} = \frac{3}{4C_{D_o}}\left(\frac{\pi AeC_{D_o}}{3}\right)^{\frac{1}{4}} . \qquad (5)$$

Substituting Eq. (5) into Eq. (4), and again using $A = b^2/S$ gives

$$R_{max} = \frac{3}{C_{D_o} 2C'}\left(\frac{\pi b^2 eC_{D_o}}{3S}\right)^{\frac{1}{4}}\left(\frac{391W_g}{\sigma S}\right)^{\frac{1}{2}}\left[1 - \left(\frac{W_f}{W_g}\right)^{\frac{1}{2}}\right] , \qquad (6)$$

$$R_{max} = \frac{1.5}{C'C_{D_o}}\left(\frac{\pi b^2 eC_{D_o}}{3S}\right)^{\frac{1}{4}}\left(\frac{391W_g}{\sigma S}\right)^{\frac{1}{2}}\left[1 - \left(\frac{W_f}{W_g}\right)^{\frac{1}{2}}\right] ,$$

where C' is the fuel consumption measured in pounds per hour per pound of thrust, and W_f is the gross weight of the aircraft at the end of its mission.

Endurance. The maximum endurance (in hours) of a turbojet aircraft, independent of altitude, is given by**

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*Equation (4) gives the maximum " ... range within 3 percent of the absolute optimum for most airplane-engine combinations." (Ref. 25, pp. 191-192.)

**Ref. 25, p. 191.

$$E_{max} = \frac{2.3}{C'}\left(\frac{C_L}{C_D}\right)_{max} \log_{10} \frac{W_g}{W_f} \quad (7)$$

Again, for parabolic polar variation of C_D versus C_L

$$\left(\frac{C_L}{C_D}\right)_{max} = \frac{1}{2}\left(\frac{\pi Ae}{C_{D_o}}\right)^{\frac{1}{2}} , \quad (8)$$

and

$$E_{max} = \frac{2.3b}{2\ C'}\left(\frac{\pi e}{SC_{D_o}}\right)^{\frac{1}{2}} \log_{10} \frac{W_g}{W_f} . \quad (9)$$

The system performance characteristics and component characteristics are summarized in Table 1. The inputs for the simulation in this example are the design equations (Eqs. (1), (3), (6), and (9)), and the probability distributions for the component characteristics.

Step. 2. Determine Subjective Probabilities

Some sample probability distributions for the component characteristics are shown in Table 2. Techniques for determination and solicitation of probability estimates are discussed in Appendix B.

In this table it has been assumed that if the initial gross weight of the aircraft is known, the final gross weight is determined uniquely.* For example, if W_g = 250,000 lb, then W_f = 130,000 lb. The purpose of this assumption is simplification. In general, for a given W_g, W_f will usually vary over some range; that is, for each possible value of W_g, there will be a probability distribution for W_f.** The same relationship is assumed for b and S.

*Initial and final weights refer to the weight of an aircraft at the beginning and end of its mission.

**See section on "Correlation Among Inputs," p. 20.

Table 1

PERFORMANCE AND COMPONENT CHARACTERISTICS FOR HYPOTHETICAL AIRCRAFT DEVELOPMENT PROJECT

		Performance Characteristics			
Component Character-istics	Units	Speed (V_{max}) mi/hr	Altitude[a] (H_{max}) ft	Range (R_{max}) mi	Endurance (E_{max}) hr
W_g	lb	x	x	x	x
W_f	lb			x	x
C_{D_o}			x		
b	ft	x	x	x	x
S	ft^2	x	x	x	x
e		x	x	x	x
T	lb	x	x		
α			x		
C'	lb/hr/lb of T			x	x

[a]Recall that the design equation gives the ambient pressure ratio at absolute ceiling. The corresponding altitude is found in a table of ambient pressure ratio versus altitude.

Table 2

PROBABILITY DISTRIBUTIONS FOR COMPONENT CHARACTERISTICS (INPUTS) IN HYPOTHETICAL AIRCRAFT DEVELOPMENT PROJECT

W_g (lb)	250,000	260,000	270,000	280,000	290,000
W_f (lb)	130,000	140,000	150,000	160,000	170,000
p(W)	.05	.10	.35	.30	.20
C_{D_o}	.015	.016	.017	.018	--
$p(C_{D_o})$	.10	.40	.30	.20	--
b (ft)	170	175	180	185	190
S (ft^2)	3,500	3,600	3,700	3,800	3,900
p(S)	.15	.15	.35	.25	.10
e	.70	.75	.80	.85	.90
p(e)	.20	.25	.35	.15	.05
T (lb)	65,000	70,000	75,000	80,000	--
p(T)	.15	.35	.25	.25	--
α	1.2		1.6	1.8	--
$p(\alpha)$	.20	.25	.30	.25	--
C'(lb/hr/lb of T)	.80	.85	.90	.95	--
p(C')	.30	.30	.25	.15	--

Step 3. Generate Probability Distributions for System Performance

The outputs from the simulation are the probability distributions for the aircraft performance characteristics. Samples of these distributions are shown in Table 3. These figures are merely representative of the type and form of the output distributions; they were not obtained by carrying out the simulation using the design equations and the information in Table 2. The process of generating probability

Table 3

PROBABILITY DISTRIBUTIONS FOR SYSTEM PERFORMANCE CHARACTERISTICS (OUTPUTS) IN HYPOTHETICAL AIRCRAFT DEVELOPMENT PROJECT

V_{max} (mph)	650	675	700	725	750
p(V)	.15	.20	.45	.10	.10
H_{max} (ft)	45,000	50,000	55,000	60,000	65,000
p(H)	.10	.20	.40	.25	.05
R_{max} (mi)	3,750	4,000	4,250	4,500	4,750
p(R)	.25	.35	.20	.10	.10
E_{max} (hr)	13.50	13.75	14.00	14.25	14.50
p(E)	.10	.15	.15	.40	.20

distributions using Monte Carlo simulation is discussed in Appendix C. The discussion in Appendix C uses a hand-calculated example. In any real application of the procedure described in this Memorandum, a computerized simulation would be required because of the size of the problem.*

Responsibility for performing the Monte Carlo simulation has some bearing on what items of information are reported to various management levels. Consider the two highest levels shown in Fig. 1. If the government agency performs the simulation, it receives the design equations and the probability distributions for the variables in the design equations from the major contractor. If the major contractor performs the simulation, the information reported to the government agency is the output of the simulation. In the latter case, the government agency may also want the input data for the simulation as an indication of the status of the subsystems and components.

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*An estimate of the computer time required for the aircraft development example is obtained as follows: The equivalent of approximately 200 multiplications is required to obtain one sample from each of the probability distributions in Table 2, and to calculate the values of the system performance characteristics. Thus 20,000 multiplications would be required for a sample of 100. On a computer with a multiply time of ten microseconds (10^{-5} sec) the total time would be one-fifth of a second.

Step 4. Calculate Statistical Measures of Status

The probability distributions for the critical system performance characteristics represent the state of knowledge regarding the system being developed. However, the data in this form do not give an unambiguous picture of status. The problem is interpretation of probability distributions--what does one distribution signify and how does it compare with another?

Probability distributions are characterized by certain statistical measures--measures of central tendency, measures of dispersion, etc. Measures of central tendency, such as the mean, median, and mode, give an indication of where the center of the distribution is located, or in the current context, a single estimate of what performance level is likely to be attained. Measures of dispersion, such as the range, variance, and standard deviation, give an indication of how widely the distribution is spread, or how much variation about the expected performance is likely. As an example, for the speed distribution in Table 3, the mean is 695 mph, and the standard deviation is 28 mph.

When required values for the performance characteristics are specified, the probability of meeting or exceeding the specification is of particular interest. If the speed specification in the example above is 700 mph, then (from Table 3) the probability of meeting or exceeding the specification is .65. Each of the statistical measures used in this Memorandum is reviewed and placed in the context of performance measurement in Appendix D. Their usage in tracking changes in status over time (measuring progress--Step 5), and their sensitivity to changes in time and cost are discussed in Sec. III. The remainder of this section deals with techniques for handling various sources of interdependence.

DEALING WITH INTERDEPENDENCIES

Probability of Meeting or Exceeding All Specifications: Correlation Among Outputs

In the aircraft development example, separate probability distributions are generated for each system performance characteristic. This

process treats the performance characteristics as being independent. When this is done, statements can be made only about meeting or exceeding each system performance characteristic independently of all the others. For example, consider the data in Table 3 and suppose that the specifications are V_r = 675 mph, H_r = 50,000 ft, R_r = 4000 mi, and E E_r = 13.75 hr. Then the probability of meeting or exceeding the speed specification is $p(V \geq V_r) = .85$, <u>independent</u> of the values of the other performance characteristics. Similarly, $p(H \geq H_r) = .90$; $p(R \geq R_r) = .75$; and $p(E \geq E_r) = .90$, all independent of each other.

If the performance characteristics above were independent, then the (joint) probability of meeting or exceeding <u>all</u> of the specifications is given by the product of the separate probabilities of (independently) meeting or exceeding the specifications:

$$p(V \geq V_r, H \geq H_r, R \geq R_r, E \geq E_r) =$$

$$p(V \geq V_r)p(H \geq H_r)p(R \geq R_r)p(E \geq E_r) = .516$$

However, the performance characteristics above are not independent and it is not valid to make a joint statement based on independently generated probability distributions. It <u>is</u> valid to make independent statements concerning the individual characteristics, and if the independent statements are satisfactory, then it is not necessary to use the slightly more complicated procedure described in the next two paragraphs.

When two, or more, system performance characteristics have one, or more, common component characteristic, the system performance characteristics are <u>not</u> independent. In the example, all system performance characteristics depend on the wing span and wing area; hence, they are correlated. When system performance characteristics are correlated, then the joint, or multivariate, probability distribution over all performance characteristics must be generated.* This is done by sampling all the component probability distributions to obtain one value for

*See Appendix B for a brief description of multivariate distributions.

each characteristic. Then values for all of the system performance characteristics are calculated.* These values describe one possible performance "configuration," or vector, for the system.

Using the previous example, the probability distributions shown in Table 2 are sampled to obtain one value for each component characteristic. Then, values for all four system performance characteristics are calculated using the design equations (Eqs. (1), (3), (6), and (9)). These values are recorded as a four-dimensional vector. A typical vector might be (V = 700 mph, H = 50,000 ft, R = 4000 mi, E = 14.00 hr). The process is repeated until a "sufficiently large" number of vectors has been generated.** The probability that the aircraft will meet or exceed all of the specifications is the ratio of the number of vectors for which $V \geq V_r$, $H \geq H_r$, $R \geq R_r$, and $E \geq E_r$ to the total number of vectors.***

Correlation Among Inputs

In addition to correlation among the outputs, there may be correlation among the input quantities (those quantities for which subjective probabilities are obtained from the engineers). This will necessarily complicate the process of determining the subjective probabilities. In some instances it may be possible to eliminate the correlation. Frequently component performance characteristics that appear to be correlated are composed of smaller components whose characteristics are independent. Because it is easier to handle probability estimates for independent characteristics, it may be worthwhile to further decompose a subsystem and thereby achieve independence at the input level.

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*The same design equations and component probability distributions are used.

**In general, a larger sample is required to obtain this type of output at the same level of confidence as in the independent case. (See footnote p. 9 and Ref. 23.)

***Calculations involving multivariate distributions become very involved as the number of variates increases. This is due to an increase in the number of arithmetical operations required. In addition, there is no single measure of variance. Instead, there is a covariance between each pair of variates. (See Ref. 23, pp. 102-103, for a brief discussion.)

In other instances, however, this may not be the preferred approach. As an example, suppose that at some time before wind tunnel tests have been concluded it is desired to obtain probability distributions for the span and area of the wing. If the geometrical shape of the wing is specified, then the span determines the area, and vice versa. If the shape is not specified, then the span does not determine the area. However, there is usually a limited range of possible shapes and the amount that the area varies as the span changes is limited by the possible shapes. Thus, the span and area are correlated.

There are three feasible courses of action in such a situation. One is to ignore the correlation and obtain independent distributions for the characteristics. This is justified when the correlation is very weak--almost no correlation at all. When the correlation is very strong, complete correlation can be assumed and one distribution obtained, as shown in the case of initial and final gross weight in the preceding example. The third option is to take explicit account of the correlation by obtaining multivariate probability distributions. The engineer is best qualified to select the course of action to be taken, based on which one produces data that best represent his evaluation of the degree of interdependence. In making his decision the engineer should recognize that the use of multivariate distributions for component characteristics introduces some complications in both solicitation techniques and sampling procedures. Procedures based on multi-stage application of techniques used with noncorrelated variables can be constructed for handling the sampling complication. These procedures are straightforward and therefore not discussed in this Memorandum. Both of these complications entail increased costs and some analysis should be made to compare the benefits of more realistic inputs to the costs of obtaining and using them.

UNCERTAINTY ARISING FROM COMPETING DESIGNS

The preceding discussion has implicitly assumed that all choices between alternative subsystem and component designs have been made. When any of the choices remain undecided they contribute to the overall uncertainty and their effect must be included. In many system

development programs these choices are purposely postponed until a configuration with a high probability of success is discovered.

In a given system development program, there may be several components for which there are competing designs. From this multiplicity of undecided design choices it can be seen that there are many possible combinations of component designs, each of which is a possible final configuration for the total system. The representation of the uncertainty with regard to choice depends on how the choices are made. The following two cases can be distinguished: (a) The choice of a design for each component may be made without consideration of the various combinations of designs for other components; in this case the choice is <u>independent</u>; and (b) The choice may be made on the basis of the characteristics of the various combinations--that is, the characteristics of the various system configurations; in this case the choice is <u>correlated</u>.

The following two schemes will yield a single set of probability distributions for the system performance: (1) For each component, combine the sets of probability distributions for the competing designs into one set. This will yield a set of probability distributions for an "average" component. These distributions and the design equations are used to generate a single set of probability distributions for the system. (2) For each combination of competing component designs--that is, for each configuration of the system--generate a set of probability distributions for the system performance. There will be as many sets of system performance distributions as there are combinations of competing component designs. These sets of distributions are then combined into one set for the system. The mechanics of effecting the combinations for both schemes can be illustrated using the following example:

> Consider a project to develop a system that has two components A and B. Suppose that each component has two competing designs, A_1 and A_2, and B_1 and B_2 and let p_{A1}, p_{A2}, p_{B1}, and p_{B2} represent their probability distributions.
>
> Under the first scheme the probability distributions for the "average" components are found by carrying out the following combinations:

$$p_{\bar{A}} = p(A_1)p_{A1} + p(A_2)p_{A2} \quad \text{and} \quad p_{\bar{B}} = p(B_1)p_{B1} + p(_{B2})p_{B2},$$

where $p(A_1)$ is the probability that design A_1 is chosen, $p(A_2)$ is the probability that design A_2 is chosen,* ..., and $p_{\bar{A}}$ and $p_{\bar{B}}$ represent the probability distributions for the "average" components. Suppose two designs for a jet engine are being pursued and the probability distributions for the thrust of the engine for each design are:

Design A_1, $p_1(T = 17{,}000 \text{ lb}) = 0.6$, $p_1(T = 18{,}000 \text{ lb}) = 0.4$;

Design A_2, $p_2(T = 17{,}000 \text{ lb}) = 0.3$, $p_2(T = 18{,}000 \text{ lb}) = 0.7$.

If the probability that Design A_1 will be selected is $p(A_1) = 0.4$ and the probability that Design A_2 will be selected is $p(A_2) = 0.6$, then the probability distribution for the thrust of the "average" component or subsystem (jet engine) is given by

$p(T = 17{,}000 \text{ lb}) = p_1(T = 17{,}000 \text{ lb}) \times p(A_1) + p_2(T = 17{,}000 \text{ lb}) \times p(A_2)$

$= 0.42$, and $p(T = 18{,}000 \text{ lb}) = p_1(T = 18{,}000 \text{ lb})$

$\times\, p(A_1) + p_2(T = 18{,}000 \text{ lb}) \times p(A_2) = 0.58.$

Probability distributions for the system performance are obtained by applying the Monte Carlo procedure using the design equations and the probability distributions for the "average" components.

The second scheme begins by using Monte Carlo to generate a set of probability distributions for the system performance for each combination of the various component designs. One set of distributions for the system is obtained by combining the sets of probability distributions corresponding to the various combinations

*Note that probabilities $p(A_1)$ and $p(A_2)$ are independent of probabilities $p(B_1)$ and $p(B_2)$. This reflects the independent choice conditions described above.

of component designs--that is, the various configurations--using the probabilities that the various combinations (configurations) are chosen. In terms of the example, there are four combinations of competing designs (A_1, B_1), (A_1, B_2), (A_2, B_1) and (A_2, B_2). The Monte Carlo procedure yields four sets of probability distributions for the system performance, one for each combination of competing designs. Denote these by p_{S11}, p_{S12}, p_{S21}, and p_{S22}. The probabilities that the four design combinations will be chosen can be represented as $p(A_1, B_1)$, $p(A_1, B_2)$, $p(A_2, B_1)$, and $p(A_2, B_2)$.* The single set of probability distributions for the system is then found by carrying out the following combination:

$$p_S = p(A_1, B_1)p_{S11} + p(A_1, B_2)p_{S12} + p(A_2, B_1)p_{S21} + p(A_2, B_2)p_{S22},$$

where p_S is the single set of probability distributions for the system.

The following points should be noted with regard to the two schemes:

(a) The first scheme can only be used under independent choice conditions. In terms of the example, the probability that either of the designs A_1 or A_2 is chosen does not depend on the probability that either of the designs B_1 or B_2 is chosen. When the purpose is to arrive at one set of probability distributions for the system, the first scheme is superior because of the shorter time required to perform a single Monte Carlo as compared to the many Monte Carlos--one for each possible design combination--with the second scheme. However, if there are correlated design choices, then the second scheme must be used.

(b) The second scheme can be used with both independent choice and correlated choice. This is because there are two ways to obtain the probabilities that the various design combinations are chosen. One way is to use the independently obtained

*As will be noted later in this section, there are two ways to obtain these probabilities.

probabilities that were used to determine the average components. In the terms of the example: $p(A_1, B_1) = p(A_1)p(B_1)$, $p(A_1, B_2) = p(A_1)p(B_2)$, etc. Obviously, this corresponds to independent choice.* This leads to the same results as obtained using the "average" components; however, the "average" component approach requires only one application of Monte Carlo.** The second way to obtain the probabilities that the various design combinations are chosen is to obtain the joint probabilities directly. In terms of the example, $p(A_1, B_1)$, $p(A_1, B_2)$, etc. are all obtained by direct questioning.

(c) In practice there probably will be some choices that are correlated and some that are not, in which case a mixture of methods will be necessary.

(d) The problem of choosing between the various configurations can be greatly facilitated by knowing the system performance probability distributions for each of the configurations. The various configurations can be compared on the basis of expected performance, or on the basis of the probability of success (meeting or exceeding the specifications). This is an avenue of further investigation that may prove to be at least as rewarding as the present effort to obtain a measure of progress. It really is a separate topic, however, and therefore will not be discussed further in this Memorandum.

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*Note that this is the definition of independence in probability theory. See Ref. 24, p. 88.

**Some difference between the two may result because of sampling differences.

III. MEASUREMENT OF PROGRESS

This section discusses the comparison of statistical measures obtained at different times as a means of measuring progress. An example is included and some comments are made about how to present the information to management. Methods for dealing with changes in contract conditions are also discussed.

TRACKING CHANGES IN UNCERTAINTY

Measuring progress at the weapon system performance level involves:

(a) Obtaining probability distributions for the critical system performance characteristics and calculating the desired statistical measures as described in the preceding section (steps 1-4); and

(b) Comparing the values obtained with the values of the same measures obtained at an earlier date (step 5).

The measures of progress obtained in this way are comparative. They indicate the relative amounts of progress made during different time periods for a given project.* This suggests that some pattern of progress may be developed that can be used as a standard.** However, in this Memorandum "tracking" means observing values of statistical measures at different times. It has nothing to do with comparing the measures to standards or budgeted figures.

Generally, progress is indicated by changes in measures of central tendency, by increases in probabilities of meeting or exceeding specifications, and by decreases in measures of dispersion. At this point, it is important to emphasize that time and cost must be fixed. The

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*It may also be possible to compare parallel projects using such measures. However, the projects must be strictly parallel as regards the time and amount of money allowed and the type of product being developed.

**Development is a process of sequential accumulation of knowledge--learning. Further research incorporating some concept of the rate of learning may be interesting (see Ref. 16, pp. 13-16).

probability distributions obtained at any time are conditional upon the time and money remaining to complete the project. If the time or the money condition is changed, then the probabilities also may change.

Referring once again to the example of a hypothetical aircraft development project, assume a minimum speed of 675 mph has been stated as a performance objective. This specification is not to be exceeded by more than 7.4 percent; i.e., the maximum acceptable speed is 725 mph.* For purposes of illustration, assume the time period for the contract is two years and the price is $100 million. Table 4 shows probability distributions, p(V), derived at 3-month intervals, for the speed of the aircraft.** The data represent the history of the development up to the final three months; thus, no data are shown for the end of the project. At the end of the final period, the aircraft will be test flown, and its speed, under anticipated operational conditions, will be known with certainty. Table 5 illustrates the summary statistics and time and cost information required to evaluate progress and status. The statistical measures in Table 5 were derived from the data in Table 4 (using the definitions in Appendix D).

The probabilities in Table 4 were chosen to illustrate the importance of using more than one of the statistical measures as an indicator of progress or status. This is apparent at three different times. First, in going from time t = 9 mo to time t = 12 mo, $\sigma(V)$ shows a slight increase. Taken by itself, this would indicate "negative" progress.***

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*Savings in cost or time are preferable to attaining a speed greater than 725 mph.

**These distributions are the output from the Monte Carlo process. The figures need not be for equal time intervals as shown.

***"Negative" progress may be an inappropriate label for such situations. Increases in the standard deviation, or an alternative measure of dispersion, can occur for a number of different reasons. In the example, there was an upward shift of the entire probability distribution. This showed up as an increase in the probabilities of higher performance capability. It also resulted in the distribution being more spread out; hence a larger dispersion. An increase in dispersion may also result from the discovery of an unforeseen problem; however, discovering this amounts to "positive" progress in a learning sense.

Table 4

PROBABILITY DISTRIBUTIONS FOR SPEED OF HYPOTHETICAL AIRCRAFT AT THREE-MONTH INTERVALS DURING DEVELOPMENT

V \ t	0 mo	3 mo	6 mo	9 mo	12 mo	15 mo	18 mo	21 mo
650 mph	.25	.20	.15	.10	.05	.05	.05	.00
675 mph	.25	.30	.30	.30	.30	.25	.20	.20
700 mph	.25	.25	.30	.35	.35	.40	.50	.55
725 mph	.25	.25	.25	.25	.25	.25	.20	.20
750 mph	.00	.00	.00	.00	.05	.05	.05	.05

Table 5

INFORMATION FOR EVALUATION OF PROGRESS AND STATUS OF HYPOTHETICAL AIRCRAFT DEVELOPMENT PROJECT SHOWN IN TABLE 4

Item \ t	0 mo	3 mo	6 mo	9 mo	12 mo	15 mo	18 mo	21 mo
$p(V \geq 675)$	.75	.80	.85	.90	.95	.95	.95	1.00
$p(V > 725)$	.00	.00	.00	.00	.05	.05	.05	.05
$\mu(V)$ (mph)	688	689	691	694	699	700	700	703
$\sigma(V)$ (mph)	29.7	28.6	26.1	25.6	26.3	25.9	22.4	19.2
Dollars consumed (millions)	Negligible	6	10	12	13	13	15	15
Time interval (mo)	0	3	3	3	3	3	3	3
Dollars remaining (millions)	100	94	84	72	59	46	31	16
Time remaining (mo)	24	21	18	15	12	9	6	3

Also, the probability of being in the acceptable range shows no progress. However, both $p(V \geq 675)$ and $\mu(V)$ increase, indicating progress. Second, from time $t = 12$ mo to time $t = 15$ mo, $p(V \geq 675)$ shows no change, but both $\mu(V)$ and $\sigma(V)$ show slight improvements. Third, from time $t = 15$ mo to time $t = 18$ mo, $p(V \geq 675)$ and $\mu(V)$ do not change, indicating no progress. However, $\sigma(V)$ does indicate progress.

It has been assumed in the example that the first time subjective probabilities are collected and the probability distributions for the system performance characteristic are generated, the amount of time and money consumed are negligible in comparison to the requirements for the total project. This will not always be the case.

The example not only illustrates the need to look at more than one measure of progress; it also indicates that the cause of difficulty cannot be determined from the summary statistical measures. Thus, the method can be thought of as a tool for management by exception. An indication of unsatisfactory progress is a signal to look deeper into the project to determine the cause. The probability distributions for the characteristics of the component parts are the first place to look. Examination of these distributions will indicate which component, or components, to analyze further.*

Graphical Presentation

The statistical measures may be displayed graphically in several ways. Figure 5 shows the probability of meeting or exceeding the minimum speed requirement (the design specification), and the probability of being within the maximum and minimum specifications. Figure 6 shows the magnitude of the standard deviation of the distributions for speed. Figure 7 shows the mean, μ, of the distributions and the mean plus and minus the standard deviation, $\mu \pm \sigma$.

In Fig. 6, for example, if the magnitude of the standard deviation in mph is taken as a measure of uncertainty, the amount of uncertainty remaining (status) can be measured at any given time. The distance from the horizontal dashed line down to the standard deviation line is the total amount of uncertainty removed. Thus, at time t = 9 mo, the amount of uncertainty removed is 4.1 mph, and the time remaining is

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*The search for underlying causes of difficulty would be facilitated by a list of factors that the individual engineers considered to be important when they were questioned to determine their probabilities. This suggests that a checklist of factors frequently considered may be an appropriate adjunct to any input sheet developed for recording engineers' subjective probabilities.

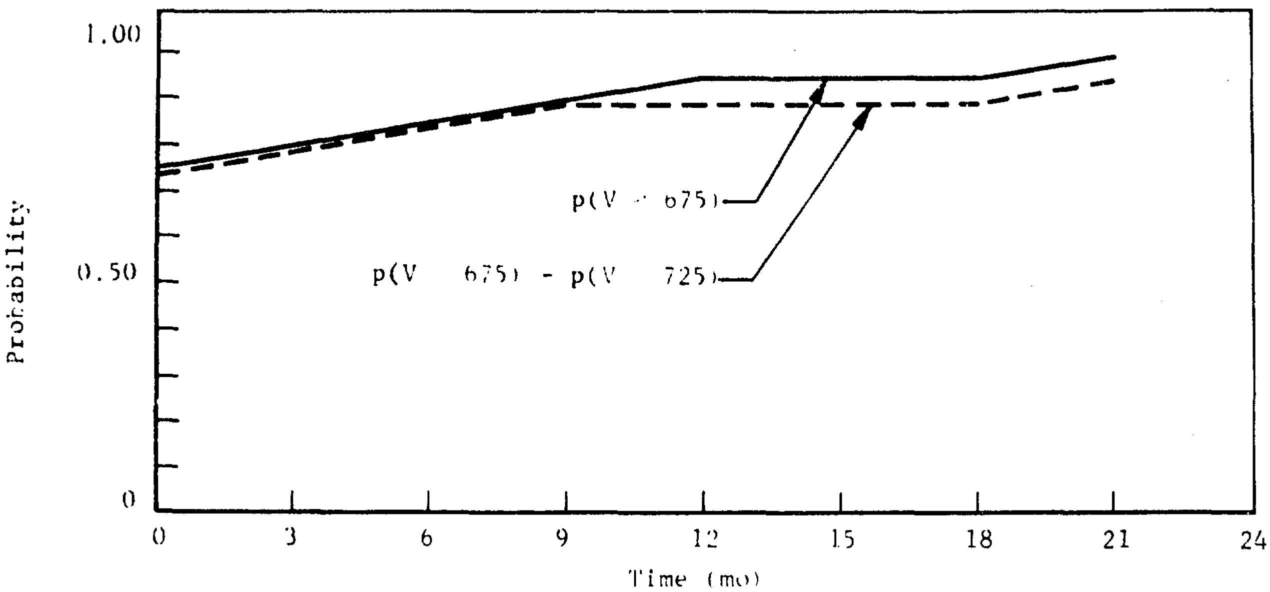

Fig. 5 - Probability of Meeting or Exceeding Minimum Requirement and Probability of Being within Maximum and Minimum Requirement Limits

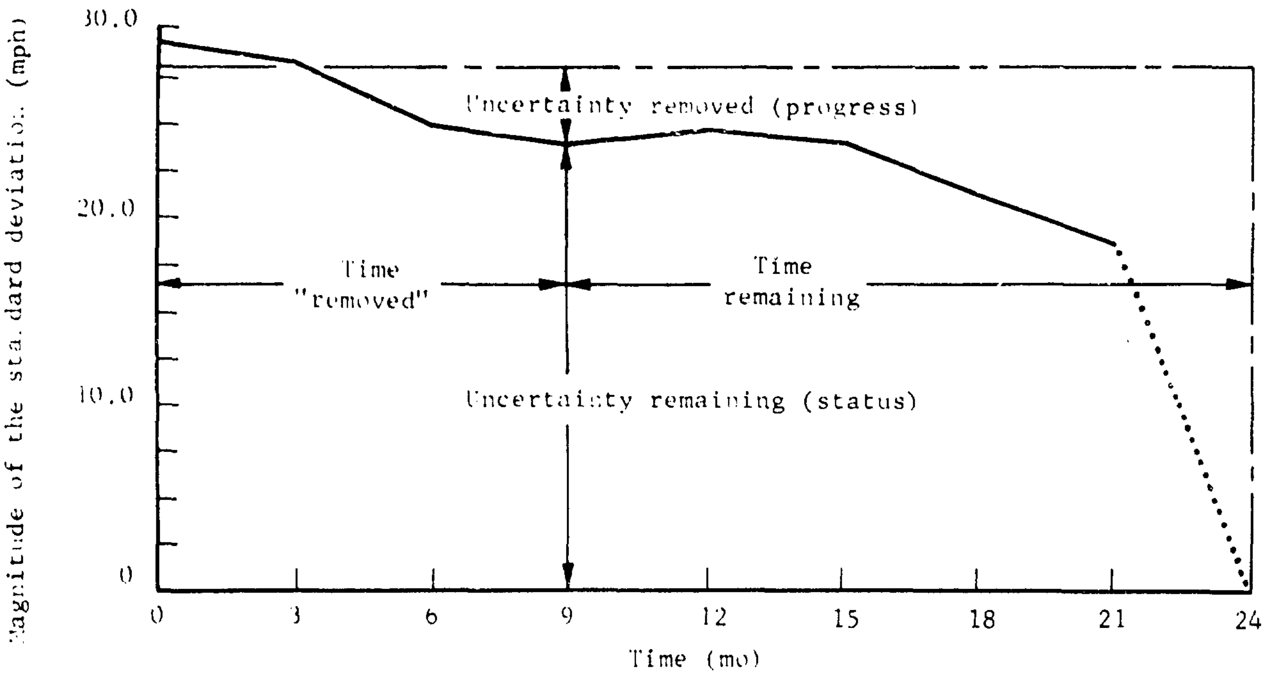

Fig. 6 - Magnitude of Standard Deviation at Three-Month Intervals

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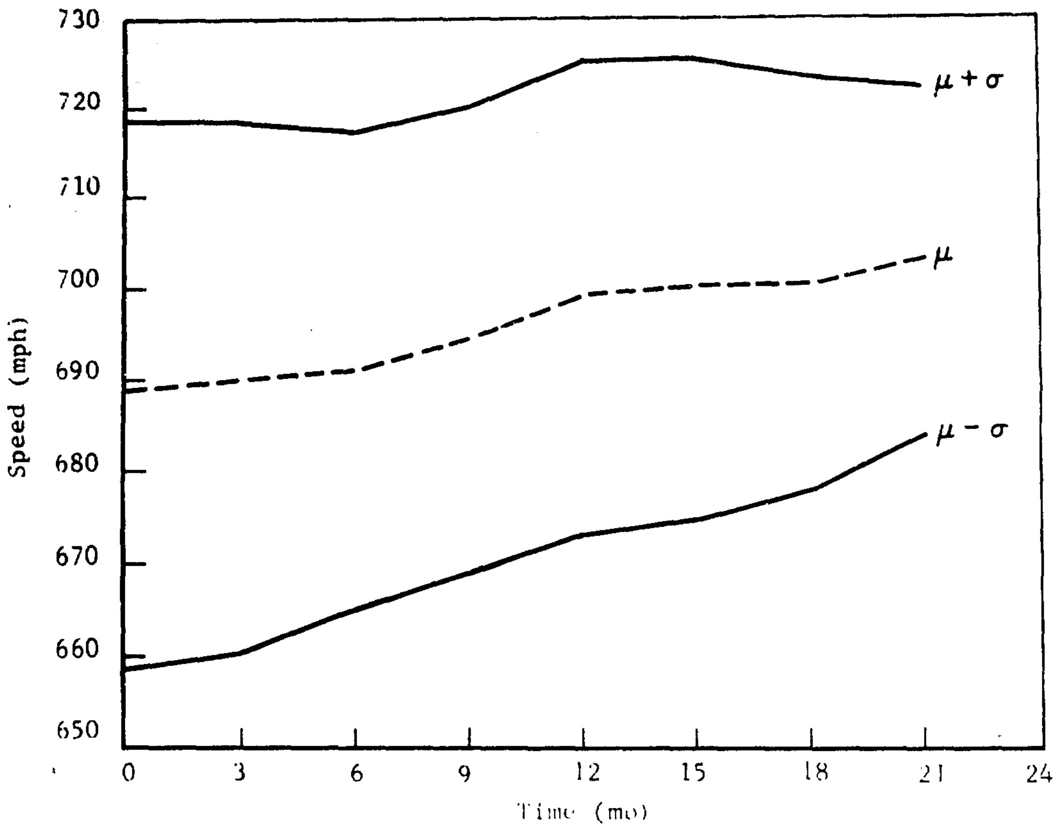

Fig. 7 - Mean Plus and Minus Standard Deviation at Three-Month Intervals

15 mo. A diagonal dotted line is shown from time 21 mo to the time axis at time t = 24 mo to indicate that after the aircraft is tested under the anticipated operational conditions there will be no uncertainty regarding its rated speed.

THE EFFECT OF CHANGES IN THE CONTRACT

In the discussion above, it has been assumed that the time, cost, and technical objectives of a development project remain fixed. This is rarely true in reality. Changes in any of the terms of a contract--time, cost, or performance*--will result in different appraisals of how

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*Changes in specifications as a source of uncertainty have been examined empirically by H. B. Eyring (see Ref. 10). His findings suggest that contractor personnel view requirements (specifications) as being flexible. Hence, there is not only uncertainty about what can be done but also about what the customer wants.

the project will turn out. This is so because the subjective probabilities obtained at any given time reflect not only the knowledge at that time but also the time and money remaining to complete the job and the management and organization of the project. In other words, the subjective probabilities depend on the conditions at the time the probabilities are obtained. If changes are made in any of the conditions, then the probabilities may also change.*

Changes in the time, cost, or specifications may be precipitated by changes in some source of uncertainty that is external to the contractor--for example, uncertainty regarding enemy capabilities.** Regardless of the source of such changes, when it is important that their impact on the measurement of status and progress be distinguished from the gains in technical knowledge made by the contractor,*** then the approach described below can be used.

Time-Cost Changes

It is assumed that changes in allocation are determined at managerial levels above the person (the working-level engineer) who provides probability estimates. Thus, the engineer is not a direct participant in achieving any changes in the time or money available. However, a change in resources will usually have an impact on potential performance and will require new probability estimates from the engineer.

The two sources of change in uncertainty can be isolated by asking the engineer for two sets of subjective probabilities.† One set

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*Both component characteristic probabilities and design choice probabilities are subject to change.

**Ref. 14, pp. 15-16.

***This distinction suggests the use of the technique described in this Memorandum as a means for comparing alternative resource allocation schemes. The government agency can select one of the alternatives on the basis of time, cost, and expected performance. This also may be done during the project to determine the effect of proposed changes in resource allocation on expected performance.

†There are arguments for and against informing the engineer of the changes in allocation before he is questioned to determine his probabilities under the old allocation. The issue is not clear. Furthermore, as a practical matter, there usually is substantial advance notice of proposed changes.

would be obtained for the old allocation pattern and the other set for the new. The probabilities associated with the old allocation yield uncertainty measures that reflect the technical knowledge gained to that time. The probabilities associated with the new allocation reflect both the technical knowledge and the change in allocation. Thus the difference between measures for the two sets of probabilities indicates the change in uncertainty associated with the change in allocation.

Continuing with the aircraft development example, suppose that at the end of 21 months the contractor is informed that he has been given an additional $30 million and 6 months to complete the project. The contractor notifies the engineers involved and a new set of subjective probability distributions is obtained. Suppose that the probability distribution for the speed of the aircraft under the new conditions is $p(V = 675 \text{ mph}) = .10$, $p(V = 700 \text{ mph}) = .60$, $p(V = 725 \text{ mph}) = .25$, and $p(V = 750 \text{ mph}) = .05$. The associated statistical measures are then $p(V \geq 675 \text{ mph}) = 1.00$, $p(V > 725 \text{ mph}) = .05$, $\mu(V) = 706$ mph, and $\sigma(V) = \pm 17.4$ mph. Figure 8 shows the "history" of the standard deviation for the project as in Fig. 6 in thin lines. The bold-face addition depicts

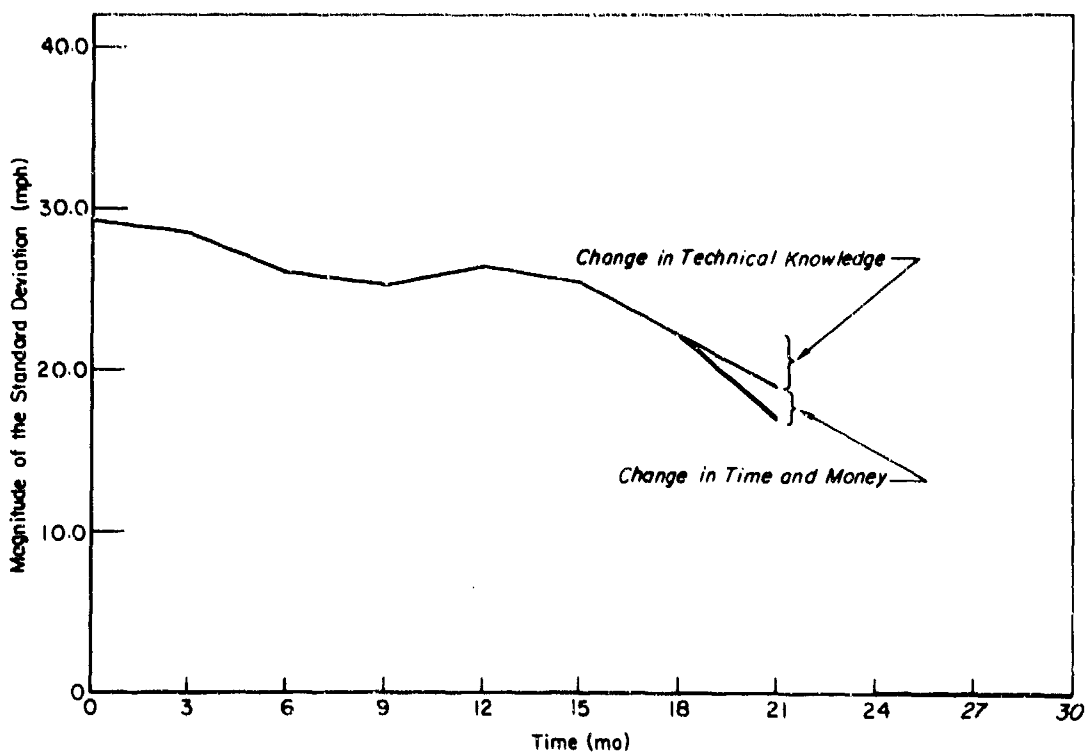

Fig. 8 - Standard Deviation Assuming a Change in Time and Money Available

the new conditions. The change in the magnitude of the standard deviation from 22.4 mph to 19.2 mph is due to the technical knowledge gained by the contractor. The additional change from 19.2 mph to 17.4 mph is due to the change (increase) in time and money available.

Specifications (Requirements) Changes

Changes in the performance requirements of a weapon system will also have an influence on the amount of uncertainty. The situation is similar to a change in the time-cost conditions. Two sets of probability distributions must be obtained. The difference between the effect of the gain in technical knowledge and the effect of the change in specifications can be determined as in the discussion of time-cost changes above.

Combined Changes

When both a specifications change and a time-cost change are made, the effect of the combined change can be separated from the effect of gains in technical knowledge, again by the same means as described above.

There may be situations in which it is desired to separate the effect of the specifications change from the effect of the time-cost change. This can be done by obtaining another set of probability distributions for one of the changes; e.g., for the change in specifications, or for the change in time-and-cost. Suppose that the specifications change is used as the basis for the third set. Then, the change due to the time-cost change is given by the difference between the new probability distributions based on both changes and the new probability distributions based on only the specifications change; the change due to the specifications change is given by the difference between the new probability distributions based on only the specifications change and the new probability distributions assuming no contract changes; and, the change due to the gain in technical knowledge is given by the difference between the new probability distributions assuming no contract changes and the old (previously obtained) probability distributions. Of course, obtaining another set of subjective

probabilities and carrying out the Monte Carlo another time requires more effort. Furthermore, there is no apparent reason for choosing between the specifications change and the time-cost change for the basis of the third set of probabilities, and there is no reason to expect that the results will be independent of the choice made.

The problems of obtaining an additional set of probability distributions and of deciding which of the change categories to choose as a basis for obtaining the third set of probabilities is eliminated if the change in specifications does not induce the contractor to make any changes.* Under this assumption, the only measures that are affected are those that relate directly to the specifications. Note that measures of central tendency and dispersion are related to the specifications only through the contractor's choice of design or management approach; hence, if the approach is not modified, then the measures of central tendency and dispersion will not be changed. The uncertainty measures that are affected are the probability of meeting or exceeding the specification and the probability of being within the maximum and minimum specification limits.

Again referring to the aircraft example, suppose that at 21 mo there are time and cost changes and also an increase in the minimum specification from 675 mph to 700 mph. The probability of meeting or exceeding the specification becomes $p(V \geq 700 \text{ mph}) = .90$. To determine the effect of the specification change only, use the probability distribution for the old allocation and subtract the probability of meeting or exceeding the new specification. For the example the change is $.80 - 1.00 = -.20$. To determine the effect of the allocation change only, next subtract the probability of meeting or exceeding the new specification using the probability distribution for the new allocation from the probability of meeting or exceeding the new specification using

*The results of this assumption are identical to the case of determining the effect of a contemplated change in specifications using the distributions based on [illegible] change and the distributions based on the time-cost change.

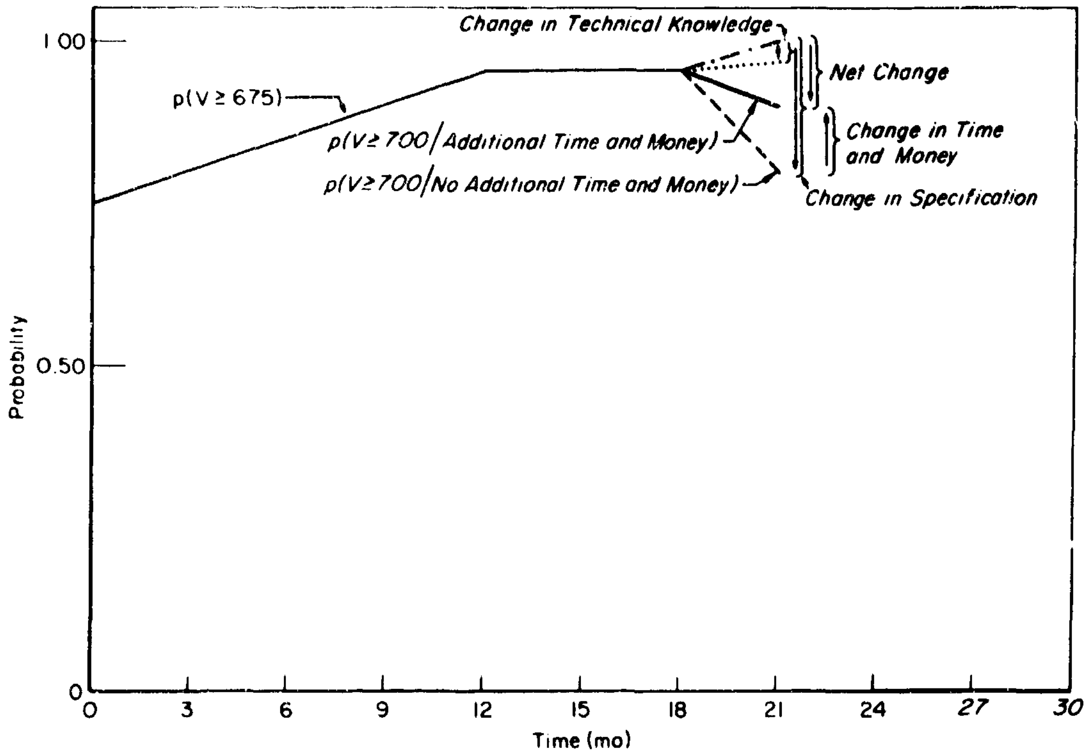

Fig. 9 - Probability of Meeting or Exceeding Specification Assuming Changes in Specification and Time and Money

the probability distribution for the old allocation. For the example, the result is .90 - .80 = +.10. The net effect is thus a decrease in the probability of success of -.10 (from 1.00 to .90).* This is shown in Fig. 9.**

In deciding whether to distinguish between gains in technical knowledge and changes in the contract the following considerations should be kept in mind:

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*In this example the two types of change are partially compensating. They can also be additive.

**The reader should note that different results for the individual changes are obtained if the order of determination is switched. This is because, in this case, the probability of meeting or exceeding the old specification is 1.00 for the old allocation. An increase in time and money will not raise this figure. In those cases where the probability of success is not already 1.00, the results will be the same for each "partial" effect regardless of the order.

(1) In order to isolate the effect of each type of change it is necessary to gather two sets of subjective probabilities and carry through two Monte Carlo simulations.* Because this involves additional effort, it is important that the recipient of the final progress data have a need for such a breakout. If he is not interested in the separate effects on the progress, then it is only necessary to obtain figures for the new conditions. (2) Additionally, during a development project there are myriad detailed engineering changes that taken individually have very small significance to system performance. If these changes were subjected to the analysis described above, almost continuous data collection and analysis would be required. Hence, it appears prudent to distinguish between change in technical knowledge and contract changes only when the contract change is a major one.

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*This is for the case when specifications changes by themselves do not induce the contractor to alter his design or management approach. When specifications changes do induce alterations and it is desired to distinguish between the effects of specifications changes and time-and-money changes in addition to the gains in technical knowledge, then the analysis and calculations become more complicated. Furthermore, the arbitrary decision about which individual contract change to consider first influences the result. Consequently, under normal circumstances such analyses probably should not be attempted.

IV. CONCLUSIONS AND SUGGESTIONS FOR OTHER APPLICATIONS

The motivation for the work described in this Memorandum has been to devise a means of obtaining better information on progress toward meeting technical performance objectives in weapon system development projects. The approach makes use of some of the important characteristics of weapon system development and decision theory. These characteristics are described in the Introduction.

The procedure that evolved uses a five-step framework. Briefly, the five steps are: (1) obtain design equations; (2) determine subjective probabilities; (3) generate probability distributions for system performance; (4) calculate statistical measures; and (5) compare measures at different times.

The first four steps lead to quantitative measures of uncertainty and figures for the probability of meeting or exceeding contract requirements (specifications). The latter quantity is a numerical statement of the chances of success. The fifth step gives a number which indicates progress, or the lack of it. The resulting numbers are indicators for management by exception. If the figures are unsatisfactory, then the probability distributions for component characteristics should be examined to discover the source, or sources, of difficulty. This search would be greatly facilitated by data regarding the factors considered by the engineers at the time of determination of probabilities.

The amount of uncertainty is influenced by the conditions of the contract--time, cost, and performance requirements. Any changes in these conditions will therefore change the uncertainty. A means of accounting for such changes has been outlined.

Implementation of such a technique should be preceded by testing of methods for determining subjective probabilities. The first application should be considered experimental. In a very real sense, the only "laboratory" that is suitable for testing the procedure is a system development project. The procedure has little utility unless it actually can be applied to such long-term, non-random processes characterized by the sequential accumulation of information.

Three other uses of the general approach are suggested for future development: (1) Comparison of the status and progress of parallel projects would provide information helpful in selection among contractors. (2) Comparison of various resource allocation schemes for one project would provide information on the chances of success for several patterns of resource allocation--information that the contractor or the government could use to evaluate and choose between the possible allocation schemes. (3) Generation of a set of distributions for the overall system for each of several alternate components would provide probabilistic information useful to the contractor or government in selecting between competing components.

Appendix A

SYSTEM DEVELOPMENT VIEWED AS A DECISION AND INFORMATION PROCESS

In weapon system development there is no unique function that determines the utility, or value, of the end item; hence, decisionmakers cannot choose courses of action that will lead to the "best" configuration for the end item. Instead, the decisionmakers must choose actions that will lead to "satisfactory" results. Decisionmaking under these conditions has been referred to by theoreticians as "satisficing,"* and the desired results are called "levels of aspiration."* The process of decisionmaking involves a comparison of results with aspirations, and a selection of a course of action. The choice depends on whether or not the comparison is satisfactory.

In weapon system development, there is periodic review. The results achieved between review points are measured by progress toward the technical objectives. The results achieved at the end of the project are values for the variables in the technical objectives. Hence, during the project, the aspiration levels must relate to how much progress is satisfactory, while at the end of the project,** the aspiration levels are concerned exclusively with the final values for the performance characteristics.

Figure 10 shows a simple satisficing model for the top management portion of the decision and information process for weapon system development. The information and decision interface between the military and a defense contractor is indicated in the figure by a dotted line. The model is presented here to show the relationship of the procedure described in the text to the military and the contractor. The procedure in the text is concerned only with boxes 10 through 13 (outlined with a heavy dashed line).

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*For example, see Ref. 30 and Ref. 11, pp. 632-634. A simple model is shown in Ref. 21, pp. 48-49.

**A more accurate distinction is between the review points when the entire system can be tested to determine values for all the variables in the technical objectives, and the points when it cannot.

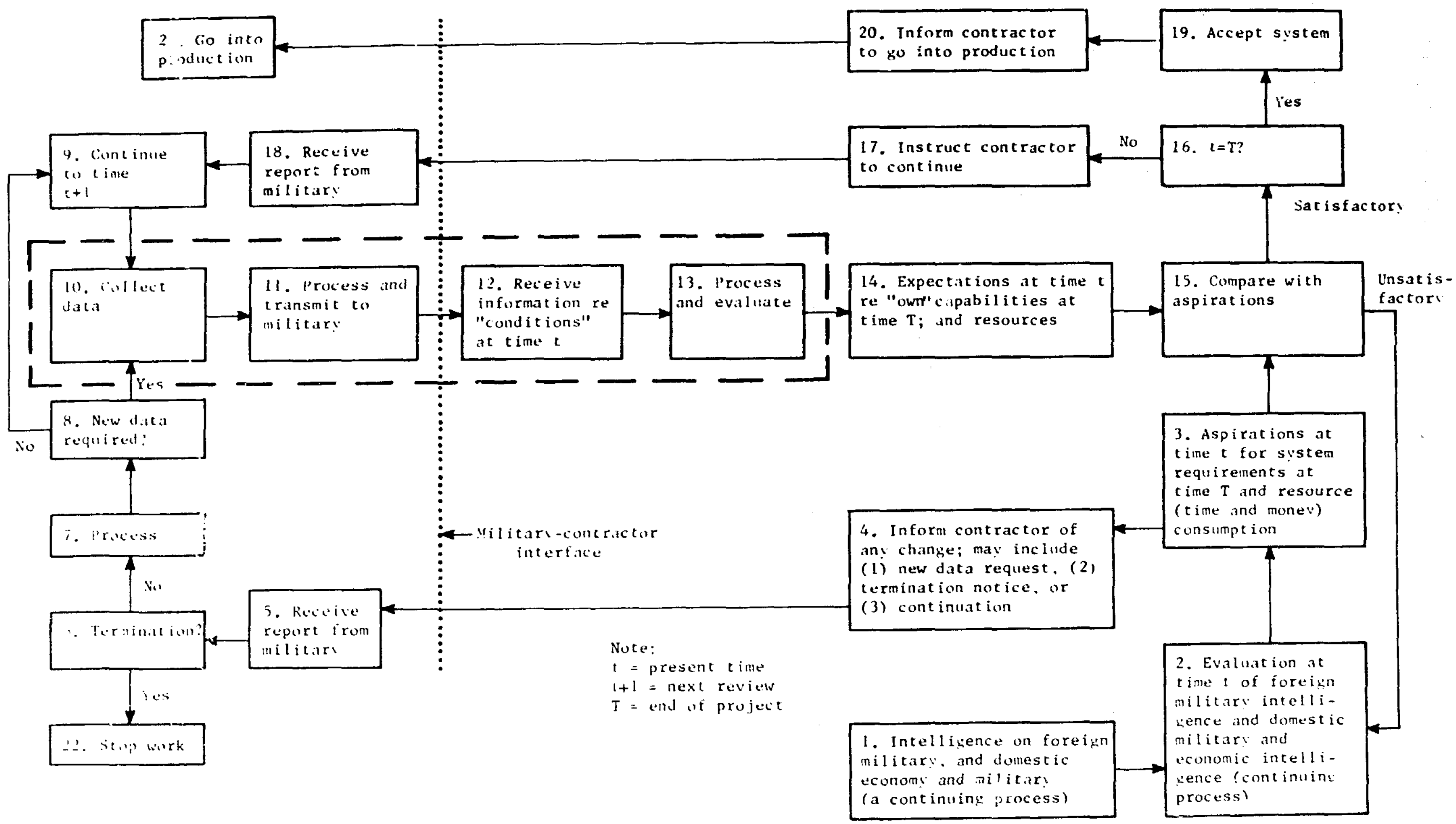

Fig. 10 - A View of the Decision and Information Process for Weapons System Development Projects

The model offers many opportunities for further development. For example, the process of setting aspiration levels (boxes 2 and 3) requires additional research. Although current decisionmaking indicates that aspiration levels exist and are used, the process by which they are set has not been thoroughly investigated.*

*See Ref. 6, Chap. 3.

Appendix B

DETERMINATION OF SUBJECTIVE PROBABILITIES

This Appendix describes the nature of subjective probabilities and a few simple methods that may be used to determine an individual's subjective probabilities. In addition, some of the factors that may influence the "quality" of the numbers obtained are discussed.*

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*The personal or subjective view of probability is relatively new. In spite of the eminent reputation and growing number of people who accept the subjective view of probability as valid, there are many intelligent and reputable scholars and practitioners who do not believe that subjective probabilities are legitimate in and of themselves; and therefore not legitimate as aids to decisionmaking. In fact, there is a heated controversy over the nature of probabilities and there are several schools of thought. (See, for example, Ref. 28.) The major issue is concerned with the nature of probabilities. The probabilities in this Memorandum are personal (individual) beliefs regarding the ultimate occurrence of an event which is in general not the result of a random process. The probabilistic school of thought which is most opposed to this personal view of probabilities is the objective school. The objectivists believe that probabilities have meaning <u>only</u> when they are related to random events, such as drawing a card from a shuffled deck, throwing a die, etc. A recent expression of the view that the use of subjective probabilities is not legitimate was given by Kenneth E. Boulding (Ref. 3, p. B-163). It is true that assignment of subjective probabilities to non-random events is a substitute for real understanding of " ... the truly systematic nature of the world ...." (Ref. 3, p. B-163.) It is also true that better decisionmaking would result if the real understanding existed. However, Boulding seems to miss the important point that many real world phenomena are so complex that even if a model could be built which reflected every detail, it would still be impossible to use the model because of the time involved in assembling the required input information and performing whatever analyses are required to get the "answer." In these cases, reliance on an exact model will be impossible. Subjective probabilities are used to capture the "best" information possible given the time constraint--whether there is an exact model or not.

Several good references on subjective probabilities are: Ref. 28, Chaps. 1, 3, and 4; Ref. 29, Chaps. 1, 3, and 12; Ref. 12, Chap. 9, especially pp. 253-263; Ref. 19, Chap. 13; Ref. 2, Chap. 1; Ref. 7; Ref. 18; and Ref. 20. The last reference is the most recent survey of subjective probabilities and decisionmaking. It contains an extensive bibliography, including publications by nearly all prominent researchers in the field.

POSSIBILITIES AND PROBABILITIES: THE NATURE OF SUBJECTIVE PROBABILITIES

Subjective probabilities are numbers that are subjective in the sense that they express the beliefs of an individual person* and probabilities in the sense that they satisfy the axioms of probability theory.**

There are many activities undertaken by man that have several possible outcomes (possibilities). When an individual must make decisions under such conditions he usually enumerates the possibilities as part of his analysis. In addition, the decisionmaker must have an assessment of the chances (probabilities) that each of the possibilities will occur. There are several common schemes for the assessment of chance. Some examples are " ... evaluative phrases (good, fair, not so hot, etc.), numerical rankings (I, II, III), alphabetical rankings (A, B, C, D), or numerical odds (1 in 5)."*** The most extensive mathematical analyses can be made if the assessment is expressed in terms of probabilities.

The statement of any subjective judgment obviously presupposes some knowledge of the subject by the individual. The source of knowledge is past experience with similar or analogous events. The knowledge may be personal or it may be obtained from the experience of others. It may be in the form of extensive statistical compilations, or it may be fragmentary bits of information. There is nearly always some information available.

Given that some information is available on which individuals can base their subjective judgments and that it is desirable to have these judgments stated in the form of probabilities, it is necessary to examine various ways to get persons to state their judgments in probabilistic terms.

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*Ref. 27, p. 3.

**Ref. 24, p. 18.

***Ref. 12, p. 255.

TECHNIQUES

Techniques for determining subjective probabilities can be identified with one of the following three categories, as distinguished by L. J. Savage:*

1. Question the person directly; ask him what probability he attaches to the outcome of a particular event. Although this is the simplest way, it is the least likely to work because many individuals do not think directly in terms of probabilities. Furthermore, those who do think in probabilities may have difficulty communicating them without the aid of some auxiliary device.

2. Observe the actions of the person whose probabilities it is desired to ascertain. This technique cannot be used in the application considered in this Memorandum for two reasons: First, it gives information after the fact, and prior information is required. Second, it does not assign numbers to the probabilities. It only indicates the event with the highest probability.

3. Ask the person to make a choice between different "betting" situations. This may be done by using techniques that will give the probability <u>density</u> function or by using techniques that will give the probability <u>distribution</u> function (see Appendix D). Which probability function is better depends on the use to which it will be put. Drawing random samples is most easily accomplished using the distribution function. Calculation of statistical measures is done using the density function.** The merits of the techniques described here are related to the subject's ability to respond easily and intelligently, and to the confidence that can be placed in his responses.***

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*Ref. 27, pp. 26-30.

**This presents no problem, however, because the density function can be obtained from the distribution function by subtraction, and the distribution function from the density function by addition.

***Confidence is higher for this third class of techniques because of the greater "ease" of responding and also because the results derive from the individual's <u>actions</u> of choosing, not from his <u>statements</u> of numbers that he believes to be probabilities.

The responses obtained must satisfy the axioms of probability theory, regardless of the technique used. If the events are mutually exclusive and collectively exhaustive,* the conditions on the responses can be summarized in the following rules:

Rule 1. The sum of the weights assigned to any set of mutually exclusive and collectively exhaustive events shall be 1.

Rule 2. The weight assigned to any event shall be a number between 0 and 1, inclusive, 0 representing complete conviction that the event will not occur and 1 representing complete conviction that it will occur.

Rule 3. If two or more mutually exclusive events are grouped into a single event, the weight attached to this single event shall be equal to the sum of the weights attached to the original events.**

Determination of the Density Function Using Choice between Gambles***

In all "choice between gambles" modes of probability determination, the person whose probabilities are being obtained (hereafter referred to as the engineer) is confronted with a choice-making situation. He is requested to choose between a wager on the outcome of the project in question (which will be called the real world gamble) and a wager on a hypothetical gamble. The hypothetical gamble involves an objective event with given objective probabilities. The real world gamble involves the outcome or outcomes of the project with unspecified probabilities. The unspecified probabilities are to be determined by inference. The probabilities of the hypothetical gamble are varied until

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*See Ref. 29, pp. 7-9, for a discussion of mutually exclusive events. If the events are not so defined, then multivariate techniques must be used.

**Ref. 29, pp. 10-13. These rules are the same as the axioms of probability theory given in Ref. 24, p. 18. It should be noted that these rules are valid for finite sample description spaces. When the sample description spaces are not finite, certain minor changes must be made. See Ref. 24, p. 21. These changes do not significantly affect the method described in the text of this Memorandum.

***Discussion of techniques other than those described here can be found in Refs. 26 and 31. Toda [31] has developed some very sophisticated techniques that may be quite useful.

the engineer is indifferent between the two gambles. At this point it is inferred that the engineer's subjective probabilities regarding the outcomes of the real world gamble are the same as the stated probabilities for the hypothetical gamble.

Consider the case of determining the probabilities of various possible values for the thrust of a jet engine that is being developed. The questioning involves presenting the engineer with a series of choices of the form shown in Table 6. To begin, the engineer is asked to state a possible value for the thrust of the completed engine. Suppose the value he gives is 36,000 lb. This value is entered in Table 6 as shown.

Table 6

A CHOICE-MAKING SITUATION TO DETERMINE THE PROBABILITY DISTRIBUTION FOR THE THRUST OF A JET ENGINE UNDER DEVELOPMENT

Real World Gamble			Hypothetical Gamble		
Consequence	\$10	\$0	Consequence	\$10	\$0
Thrust	36,000 lb $\pm$ 1000 lb	Anything else	Event	X	Y
Probabilities	?	?	Probabilities	.7	.3

Because the final distribution that will be obtained by the questioning will be discrete,* the real world events must be viewed as having some range of possible values, say 36,000 lb $\pm$ 1000 lb. This distinction will not enter directly into the use of the probability distributions that are obtained; however, it is related to the engineer's ability to discriminate.

The questioning proceeds as follows:** The engineer is asked to choose between the gambles shown in Table 6. If he chooses the real world gamble, then he will get \$10 if the thrust turns out to be

*All techniques described in this Appendix yield discrete probability distributions. It is possible to obtain continuous subjective probability distributions but the techniques require that the subject be able to evaluate the entire distribution at one time. This is not as easy as dealing with the distribution in pieces, and, consequently, continuous distributions are not discussed. Continuous and discrete distributions are described in Ref. 24.

**The discussion follows Ref. 12, pp. 256-258.

36,000 lb (± 1000 lb) and $0 if the thrust is anything else.* If he chooses the hypothetical gamble, he will get $10 if event X occurs, and $0 if event Y occurs.**

If the engineer prefers the hypothetical gamble, it is inferred that for him the probability that the engine thrust will be 36,000 lb is less than .7. Hence, he is offered a new choice, with a lower probability of event X. Let the new choice be as shown in Table 7. Suppose

Table 7

A SECOND CHOICE-MAKING SITUATION TO DETERMINE THE PROBABILITY DISTRIBUTION FOR THE THRUST OF A JET ENGINE UNDER DEVELOPMENT

Real World Gamble			Hypothetical Gamble		
Consequence	$10	$0	Consequence	$10	$0
Thrust	36,000 lb ± 1000 lb	Anything else	Event	X	Y
Probabilities		?	Probabilities	.6	.4

that the engineer is indifferent between these two gambles. Then it is inferred that his subjective probability that the thrust will be 36,000 lb at the completion of the project is .6 an hat his probability that the thrust will be anything else is .4.

Having obtained the probability for the thrust being 36,000 lb, the value for the thrust in the real world gamble is changed. There may be some difficulty in establishing the next highest, or lowest, value of the thrust for which the engineer can state a probability. The

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*The $10 and $0 consequences are used so the engineer will be able to discriminate on a basis of personal value. Any two consequences can be used as long as they are the same two consequences for both gambles. It is not necessary that the engineer actually receive the consequences. They are merely part of a construction that enables the engineer to discriminate and evaluate.

**A number of abstract constructions for the hypothetical gamble can be used to facilitate the questioning. For example, event X can be thought of as drawing a black ball from a bowl containing seven black balls and three white balls. Event Y is then the drawing of a white ball. Similarly, the gamble can be visualized in terms of a lottery, a spinning pointer, or a ten-sided die.

reason for this possible difficulty is that the engineer's ability to discriminate between values of the thrust is limited. Suppose that the engineer attempts to determine his probability for the thrust being 37,000 lb, and he cannot, or he gets the same results as for 36,000 lb. This indicates that the engineer cannot discriminate between 36,000 lb and 37,000 lb. When the engineer cannot discriminate, the thrust must be changed until some results can be determined.

The end-points of the distribution are determined when the value of the thrust reaches points for which the engineer is indifferent between the gambles, with the probability of event X being 0 and the probability of event Y being 1.0. Table 8 shows some hypothetical results for the procedure outlined. The end-points are at 34,000 lb and

Table 8

PROBABILITY DISTRIBUTION FOR THE THRUST OF AN HYPOTHETICAL JET ENGINE UNDER DEVELOPMENT

Thrust (lb)	Probability
34,000	0.0
36,000	0.6
38,000	0.2
40,000	0.1
42,000	0.1
44,000	0.0

44,000 lb. It is important to note that if the probability of 44,000 lb is 0, then the probability of anything higher will also be 0. Thus, accurate determination of the end-points requires that the questioning procedure begin with some value of the thrust that lies within the probability distribution and then proceeds in an orderly fashion to both higher and lower values of the thrust.

Note also that the probabilities in Table 8 satisfy the three rules given on page 43 above. If the conditions specified by the rules are not satisfied, then the probabilities must be normalized or redetermined.

The probabilities shown in Table 8 reflect the engineer's beliefs <u>at a particular time</u>. At some later time, his probabilities will change because of the additional information gained in the interim.

Determination of the Distribution Function
Using Choice between Bets

The definition of the real world gamble in Tables 6 and 7 may not be in a form that is easily comprehended by the engineer. The form requires him to think in terms of a probability density function, and it may be easier for him to think in terms of distribution functions or, more precisely, in terms of the areas under certain portions of the density functions.

The simplest type of technique for determining distribution functions uses a series of choices of the form shown in Fig. 11. There are only two events, E_1 and E_2, and two bets to choose between. Each event has a probability of .5. This situation is easy for an individual to comprehend, and it is probably the most common. (More people have encountered coin-tossing than any other random process having "known" probabilities.)

The procedure involves varying the value of T_1 until the engineer is indifferent between b_1 and b_2. Then it is inferred that the probability that the thrust will be less than T_1 is .5 and the probability that the thrust will be greater than or equal to T_1 is .5. Thus, T_1 divides the probability density function in half.

After establishing T_1, the questioning moves down one level as shown in Fig. 10. At this level, the two halves of the probability density function are halved. The values T_2, T_1, and T_2' divide the probability density function into fourths; that is, the probability that the thrust will be less than T_2 is one-fourth, the probability that it will be greater than or equal to T_2 and less than T_1 is one-fourth, etc. The process is continued until the engineer can go no further. As an illustration, suppose that the engineer is successful in responding at the third level but no lower. Some hypothetical results are shown in Table 9, and the corresponding probability distribution function in Table 10. Two adjustments must be made before the distribution function can be used for sampling. First, the "ends" of the distribution must be determined. Second, the values of thrust to be used in the sampling must be determined.

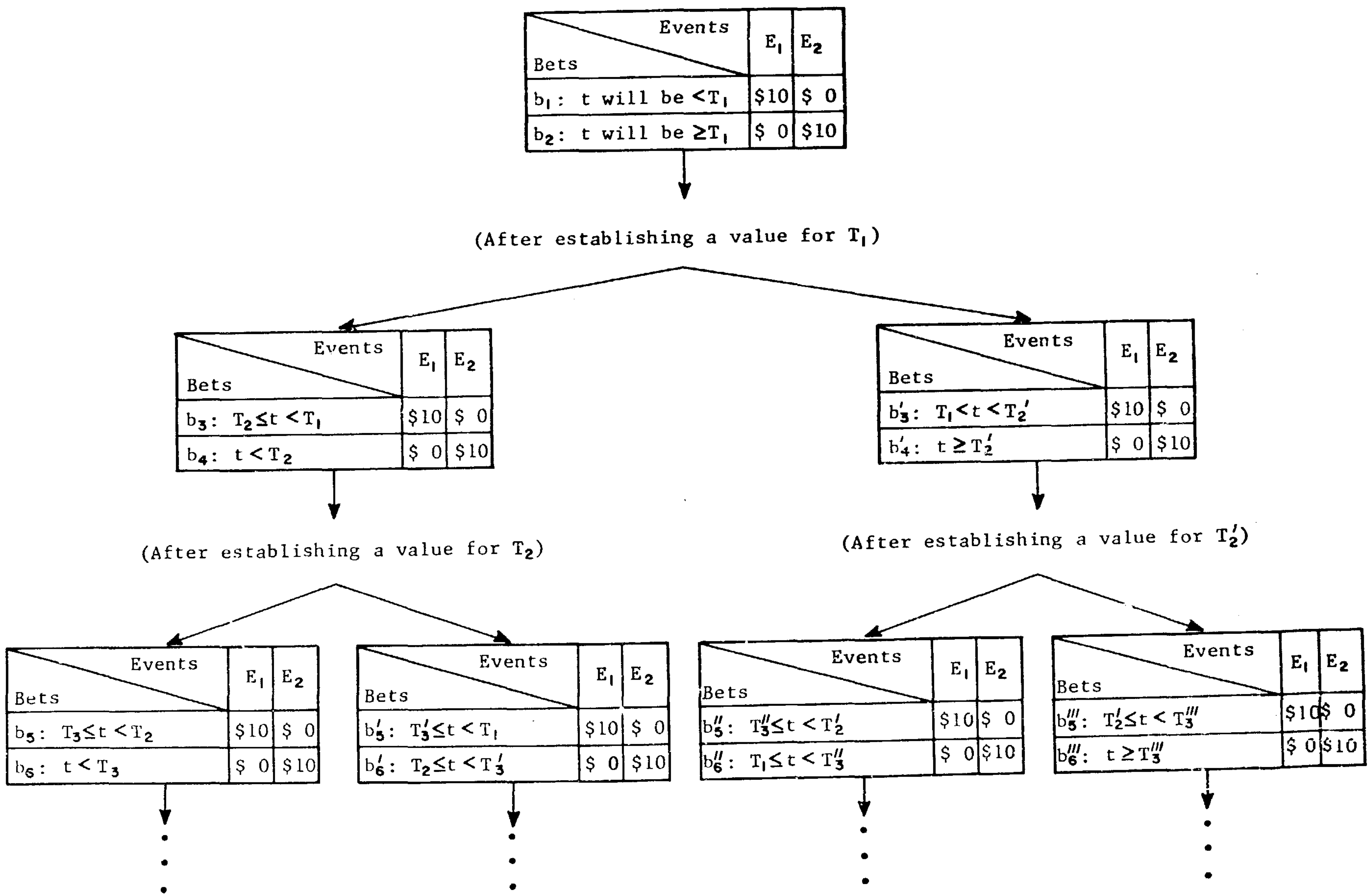

Fig. II – Series of Choices Between Bets to Determine a Probability Distribution Function

Table 9

HYPOTHETICAL RESULTS OF QUESTIONING PROCEDURE ILLUSTRATED IN FIG. 10

T_i	Value (lb)
T_1	38,000
T_2	36,000
$T_{2'}$	40,000
T_3	35,500
$T_{3'}$	37,000
$T_{3''}$	39,500
$T_{3''''}$	40,500

Table 10

PROBABILITY DISTRIBUTION FUNCTION CORRESPONDING TO RESULTS SHOWN IN TABLE 9

T_i	$p(t \le T_i)$
35,500	.125
36,000	.250
37,000	.375
38,000	.500
39,500	.625
40,000	.750
40,500	.875

The highest and lowest values of the thrust shown in Tables 9 and 10 do not determine the end-points of the distribution. The lowest value is 35,000 lb. The probability that the thrust will be lower than this value is 0.125. However, there is no indication of how much lower the thrust may be. The same is true for the other end of the distribution. To determine the ends, the procedure described in the previous section is used. The lower end-point, T_ℓ, satisfies the condition that it is the largest value of the thrust for which $p(t > T_\ell) = 1.0$. The upper end-point, T_u, satisfies the condition that it is the smallest value for which $p(t \leq T_u) = 1.0$ Table 11 shows the distribution function with end-points.

Table 11

PROBABILITY DISTRIBUTION FUNCTION
OF TABLE 10 WITH END-POINTS

T	$p(t \leq T)$
35,000	.000
35,500	.125
36,000	.250
37,000	.375
38,000	.500
39,500	.625
40,000	.750
40,500	.875
41,000	1.000

Some adjustments must be made in the distribution function shown in Table 11 to put it into a suitable form for sampling. The probability that the thrust will be between 35,000 lb and 35,500 lb is 0.125. For the purpose of calculation, either 35,000 or 35,500 or some number in between must be selected. The simplest choice is to select the mid-point, 35,250. This is shown by a dashed line in Fig. 12. The mid-points for all other values are also indicated by dashed lines.

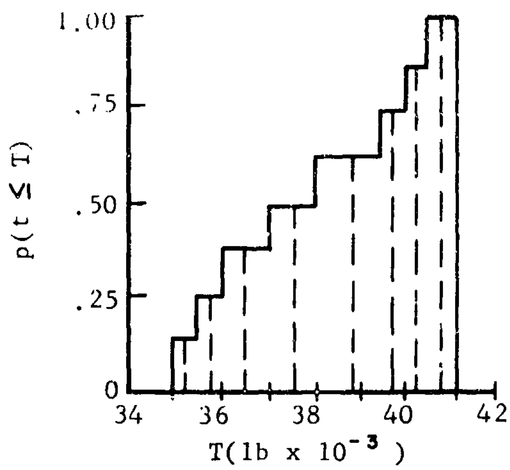

Fig. 12 - Distribution Function for Table II

The Standard Lottery

The standard lottery is a technique for determining the probability density function.*

> Suppose that [an engineer is] offered a free chance at a prize of value V under the following conditions. Balls numbered 1 to 100 have been placed in an urn and one of these balls has then been drawn and put in a closed box. [He is] presented with 100 tickets numbered from 1 to 100 and [is] allowed to choose one of them. If the number [he chooses] matches the number on the ball which has been drawn from the urn, [he] will receive the prize; if not, [he receives] nothing. Suppose further that even though the prize is one which [he is] extremely anxious to win, [he does] not feel that it is worth the slightest effort to look for a ticket with any particular number on it; [he] simply [takes] the first one which comes to hand.
>
> In such a situation [it is said that in his] opinion the 100 possible events are equally likely. Notice very carefully that [it is not possible to] "prove" that the events are "in fact" equally likely: the fact is that the ball which has been drawn has some one particular number and no other. But

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*The description in this section is excerpted from Schlaifer (see Ref. 29, pp. 13-14). Minor changes have been made to adapt his discussion to the context of the jet engine development used to illustrate the preceding techniques. To facilitate this conversion, the events he refers to have been replaced by the values of the thrust shown in Table 8. Schlaifer uses the word "prize" to denote the same thing as "consequence."

even though anyone who knew which ball has been drawn would not be indifferent among the 100 tickets, [the engineer's] decisions must be based on what [he knows or believes] about the facts of the world--they cannot be based on the unknown truth about these facts. Therefore if [he is] indifferent in the way described, then for [him] the 100 events are equally likely by definition.

Now if [the engineer's] state of mind as just described is to be described by numerical weights attached to the 100 possible events 1, 2, ..., 100, it is clear that these weights must all be equal. If the sum of these 100 equal numbers is to be 1, as required by Rule 1, it is also clear that the number attached to each event must be 1/100. Rule 3 then tells us that events such as "ball number 2 or 7" must have weight 1/100 + 1/100 = 2/100, that events such as "any ball numbered between 1 and 37 inclusive" must have weight 37/100, and so forth. Thus while Rule 2 specified only that the weight attached to any event must be a number between 0 and 1 inclusive, [the standard lottery describes] a way of selecting a specific number within this range to describe [a person's] attitude toward any conceivable event in this lottery.

What is more important, [an engineer] can find the unique set of weights which describes his attitudes in a more complex situation by using a lottery of this sort as a standard of comparison. In order to decide what weight to assign to the event ["thrust = 36,000 lb," in Table 8], the [engineer] can imagine that he is given a choice between a certain number of tickets in the standard lottery with a prize of value V as described above and the right to receive this same prize to the event ["thrust = 36,000 lb"]. If in his opinion the right to receive this prize in the event of ["thrust = 36,000 lb"] has exactly the same value as [60] tickets in the standard lottery, then by definition he considers these two events equally likely and he should assign weight [60/100] to the event ["thrust = 36,000 lb"]. (It goes without saying that if the standard lottery with 100 balls does not offer a fine enough division, the [engineer] can substitute a similar lottery with more balls. If he feels that the right to receive the prize in case of ["thrust = 36,000 lb"] is worth more than [60] tickets but less than [61] in a lottery with 100 equally likely events, he may decide that it is equivalent to [605] tickets in a lottery with 1000 equally likely events.)

Having assigned a weight to the event ["thrust = 36,000 lb"] the [engineer] can proceed in the same way to assign weights to all the other events in Table [8]. These weights must of course be such that their total is 1, and therefore what the [engineer] is really doing is placing the set of collectively exhaustive and mutually exclusive events shown in Table [8] into one-to-one correspondence with a set of collectively exhaustive and mutually exclusive events in the standard lottery. When he is through, the event ["thrust = 36,000 lb"]

> will correspond, say, to the event "ball numbered between 1 and [60] inclusive," the event ["thrust = 38,000 lb"] to balls [61 to 80], and so forth. It is perhaps worth remarking that [it is] in no sense [assumed] that [an engineer] will actually be as ready to gamble on balls drawn from an urn as to make decisions concerning his regular business. We are simply assuming that a rational person can with practice <u>think abstractly about his feelings of certainty and uncertainty in any given situation</u>, regardless of any feelings he may have about any other aspects of the situation.

Determination of Multivariate Probabilities

When subjective probabilities are required for a number of variables that are correlated,* the information must be gathered as a multivariate, or joint, probability distribution. A multivariate distribution** is a probability distribution over the possible outcomes of several variables. In the case of two variables, X_1 and X_2, the distribution is of the form $p(x_1, x_2)$, where x_1 and x_2 are specific values of the variables X_1 and X_2, respectively.

There appear to be no papers published regarding the determination of multivariate, subjective probabilities. However, the techniques described above can be extended to the task. Consider, for example, the case of determining a bivariate distribution. The standard lottery can be used by having the subject distribute the tickets over an array of pairs of values for the two variables. The choice between gambles and the choice between bets techniques can be applied by fixing the value of one variable and running through the questioning procedure for possible values of the other variable. Then the value of the first variable is changed again, and so on, until the distribution is determined.

A difficulty arises because, as the number of variables increases, the subject's ability to give meaningful responses becomes increasingly impaired. This is due to the increase in the number of dimensions that must be constantly kept in mind. Hence, some experimentation is required before the usefulness of multivariate subjective probabilities can be determined.

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*This is the case as described in the text under "Correlation among Inputs," p. 20.

**See Ref. 24, pp. 285-292, for a more thorough discussion of multivariate (joint) probabilities.

SELECTION OF PROBABILITY APPRAISERS

Use of subjective probabilities as guides for decisionmaking demands careful selection of probability appraisers. The main concern is the "quality" of the responses. Several studies have been made of people's ability to estimate the probabilities of random events.* Such events have objective probabilities, and in these studies it was possible to compare the subjective probabilities and the "true" probabilities.** However, when the events being considered are not the results of random processes, it is not possible to verify the accuracy of the responses because there are no "true" probabilities. The impossibility of evaluating subjective probabilities for non-random events forces attention to evaluating the ability of people to give "meaningful" responses.

The evaluation and selection of probability appraisers is a qualitative subject. At present there are no quantitative tests of an individual's ability to express "good" subjective probabilities.*** As mentioned above, it may be desirable to conduct tests such as those reported in Ref. 20, pp. 321-327. However, there are many other factors which have a bearing on an individual's ability as a probability appraiser. The best that can be done here is to mention them.

The factors which have some bearing on an individual's ability to give "meaningful" probability appraisals can be classified as being either personal or environmental. Personal factors include intelligence, knowledge, and factors that relate to the possibility of bias, both intentional and unintentional. For example, a person of high integrity will be less likely to bias his responses than a person of low integrity.

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*Ref. 20, pp. 321-327.

**Most of the studies indicated that people overestimate low probabilities (less than approximately .20), and underestimate high probabilities (greater than approximately .20). This phenomenon may need to be compensated for when subjective probabilities are used in decisionmaking.

***A scheme has been proposed by De Finetti (see Ref. 8) for evaluating individual probability appraising skills on the basis of past performance.

Unintentional bias may be introduced by pessimism or optimism. Measurement of these factors is difficult if not impossible. Until adequate psychological tests can be developed (if they ever can), personal, common-sense appraisals of individuals, using records of past performance and other personal factors, will have to suffice.

Environmental factors include influences that are external to the individual. The amount of bias resulting from these influences depends on the interaction between the personal factors and the environmental factors. Examples of environmental factors are: specific requirements given in the contract, company aspirations, the management position of the individual, and the punishment and reward structure. Contract requirements and company aspirations may influence an individual to bias his responses to please his superiors. An individual's position in the management structure, and the punishment and reward structure interact with the individual's personal values and integrity. This is probably what is meant by an individual's "degree of involvement."

In spite of the numerous adverse influences that can introduce bias in the case of a responsible engineer, in most cases he will be the best source of information. What must be done is to develop techniques that increase the objectivity of the responses.

Discussions of methods for increasing the objectivity of subjective probabilities have concentrated on the elimination of bias. The two approaches that have received the most attention are rewarding "good" performance,[*] and comparing and combining the probabilities obtained from two or more individuals.[**] Another device that may lead to improved results is to have the probability appraiser prepare a list of factors that he considered when he determined his probabilities. Such a list could be of value when comparing probabilities obtained from different people and probabilities obtained from a single person at different times.

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[*] Ref. 12, p. 261.

[**] Ref. 12, p. 262; Ref. 9; and Ref. 13, pp. 16-18.

Appendix C

GENERATING PROBABILITY DISTRIBUTIONS USING MONTE CARLO SIMULATION

This Appendix describes the process of generating a probability distribution using Monte Carlo simulation. The example used is that of generating a probability distribution for the life of a two-component electronic device, based on the known probability distributions for the operating lives of the components (tubes), and the design equation which reflects the behavior of the device in terms of the behavior of its components. The "design equation" in this case states that the device fails if either one of its components fails. An example more closely related to the aircraft project used throughout this Memorandum is that of determining the probability distribution for the possible values of the aspect ratio of an aircraft, given the probability distributions for the wing span and the wing area. The example of an electronic device was chosen because the mathematical calculations required are simpler.* However, the equivalence of these two examples is demonstrated by noting that they both are of the form: Given $p_1(x_1)$, $p_2(x_2)$, and $y = f(x_1, x_2)$, determine $p(y)$.**

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*The electronic device example requires a comparison of only two numbers and selection of the smaller, whereas the aspect ratio example requires the squaring of one number and division of that by a second number. In an operational situation, the simulation would be performed by computer, using available Monte Carlo programs, in which case the more extensive calculations would be accomplished with a minimum of effort.

**In the electronic device example $p_1(x_1)$ and $p_2(x_2)$ are the probability distributions of the operating lives of the components; y is the the life of the device; $y = f(x_1, x_2) = \min(x_1, x_2)$; and $p(y)$ is the probability distribution of the operating life of the device. In the aspect ratio example, $p_1(x_1)$ and $p_2(x_2)$ are the probability distributions for the wing span and the wing area, respectively; y is the aspect ratio; $y = f(x_1, x_2) = x_1^2/x_2$; and $p(y)$ is the probability distribution over the possible values of the aspect ratio. This formulation can be extended to any number of characteristics as follows: Let $x_1, \ldots, x_n$ denote

Suppose that the life characteristics of two electronic tubes are as shown in Figs. 13 and 14. To use the Monte Carlo technique, these curves must be changed from density functions to distribution functions.* This is done in Figs. 15 and 16. Values along the probability scale of the distribution function are selected by means of a table of random numbers. In the case shown, the digits 0 to 9 can be used. (In cases where the probabilities involve two digits, pairs of digits must be selected from a random number table.) When a random number is selected, a horizontal line is drawn from the Y-axis on the cumulative probability curve until it hits one of the vertical lines. This determines a value for the life of the tube. For example, if the random number is 8, then the life of Tube No. 1 is 260 hr. To facilitate working out this example, the data in Figs. 15 and 16 are converted into tabular form in Tables 12 and 13.

To determine a sample lifetime for the electronic device, a sample lifetime is determined for each tube and the shortest is the lifetime for the device. Suppose the sample lifetime for Tube No. 1 is 260 hr,

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the component characteristics of some system and $y_1, \ldots, y_m$ denote the system performance characteristics. The problem is then: Given the probability distributions $p_1(x_1), \ldots, p_n(x_n)$, and the design equations $y_1 = f_1(x_1, \ldots, x_n), \ldots, y_m = f_m(x_1, \ldots, x_n)$, determine the probability distributions $p_{n+1}(y_1), \ldots, p_{n+m}(y_m)$. The aircraft example in Sec. II can be stated in these terms by making the following identifications: $x_1 = W_g$, $x_2 = W_f$, $x_3 = C_{D_0}$, $x_4 = C_{D_f}$, $x_5 = b$, $x_6 = S$, $x_7 = e$, $x_8 = T$, $x_9 = \alpha$, $x_{10} = C'$, $y_1 = V_{max}$, $y_2 = P_{ac}$, $y_3 = R_{max}$, $y_4 = E_{max}$. Then the $p_i(x_i)$ are given in Table 2; the design equations are Eqs. (1), (3), (6), and (9); and the $p_j(y_j)$ are given in Table 3.

*See Ref. 5, pp. 179-183, for further discussion. The distribution function shows the cumulative probability. The term cumulative probability is used to indicate a particular value of the distribution function.

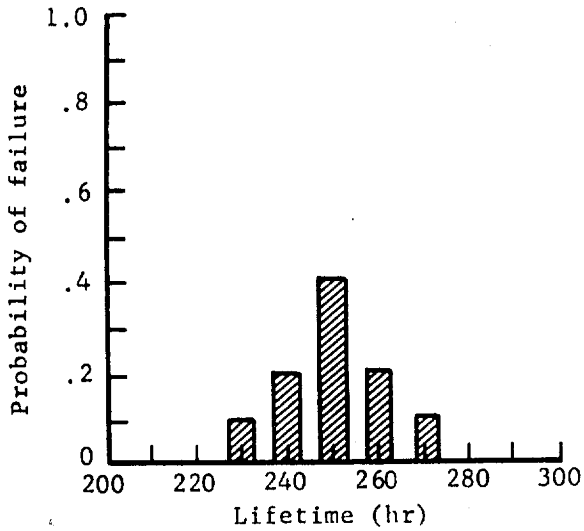

Fig. 13 - Life Curve for Tube No. 1

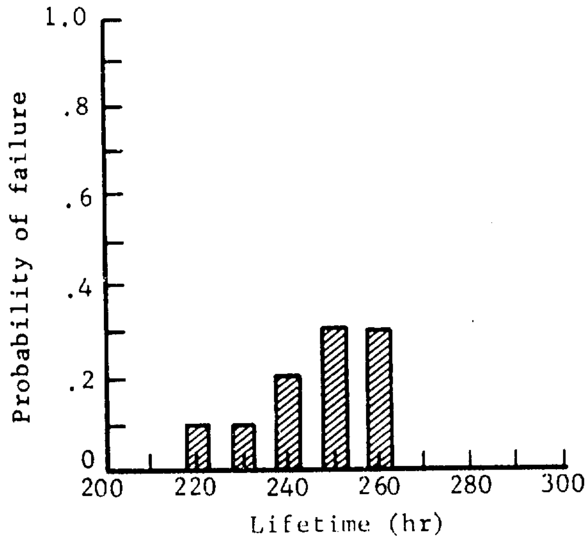

Fig. 14 - Life Curve for Tube No. 2

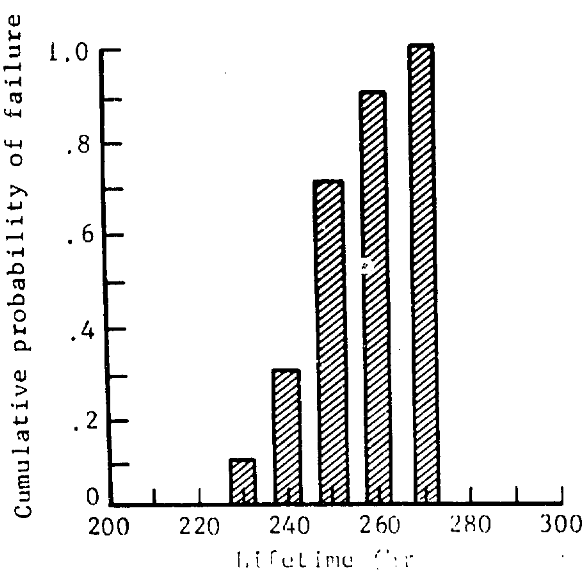

Fig. 15 - Cumulative Probability of Failure for Tube No. 1

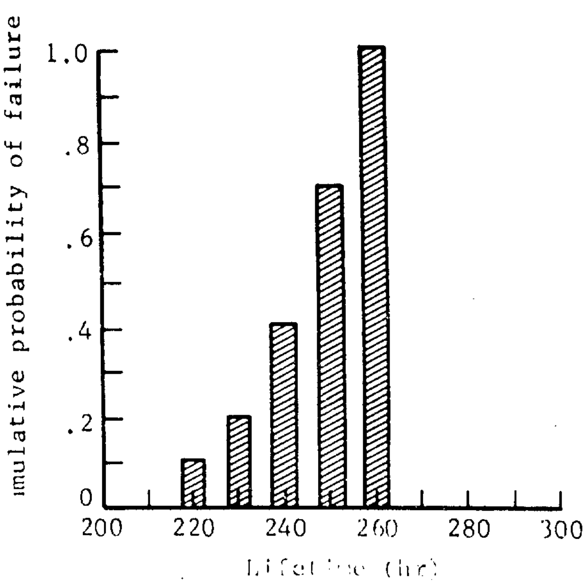

Fig. 16 - Cumulative Probability of Failure for Tube No. 2

Table 12

SAMPLING DISTRIBUTION FOR TUBE NO. 1

Random Number	Corresponding Life of Tube No. 1
9	270 hr
8-7	260
6-3	250
2-1	240
0	230

Table 13

SAMPLING DISTRIBUTION FOR TUBE NO. 2

Random Number	Corresponding Life of Tube No. 2
9-7	260 hr
6-4	250
3-2	240
1	230
0	220

as determined above. Another random number is drawn for Tube No. 2. Suppose it is 3; the lifetime of Tube No. 2 is then 240 hr. In this instance, the lifetime of the device is 240 hr.

To determine the distribution of lifetimes for the device, the above process is repeated a large number of times. The frequencies of the observed lifetimes of the device are plotted as a bar chart. This results in a life curve for the device. The number of times that this process is repeated depends on the desired accuracy. The larger the number, the more accurate the resulting life curve will be. Accuracy requirements vary among applications. In all cases, the sample size is determined using standard statistical techniques and will not be discussed in this Memorandum.*

For illustrative purposes, the life curve of the two-tube electronic device is determined by obtaining a sample of 25 pairs of lifetimes for the two tubes. First, a table of the form of Table 14 is set up.

*See Ref. 29, pp. 157-159, 275-277.

Table 14

TWENTY-FIVE SAMPLES OF THE LIFETIME FOR A TWO-TUBE ELECTRONIC DEVICE

Tube No. 1		Tube No. 2		Device Lifetime
Random Number[a]	Lifetime (from Table 12)	Random Number	Lifetime (from Table 13)	
0	230 hr	9	260 hr	230 hr
5	250	4	250	250
4	250	2	240	240
0	230	1	230	230
8	260	0	220	220
0	230	6	250	230
0	230	6	250	230
2	240	6	250	240
5	250	7	260	250
7	260	9	260	260
5	250	2	240	240
8	260	0	220	220
4	250	5	250	250
6	250	8	260	250
5	250	9	260	250
4	250	8	260	250
1	240	2	240	240
3	250	5	250	250
9	270	1	230	230
8	260	9	260	260
4	250	9	260	250
3	250	3	240	240
1	240	0	220	220
5	250	5	250	250
6	250	0	220	220

[a]Random numbers taken from Table 7-3 of Ref. 5.

Then random numbers, 25 for each tube, are obtained from Tables 12 and 13. The lifetime of the device is the lifetime of the first tube to fail. A table of the frequencies of the various lifetimes of the device is constructed (Table 15).

Table 15

LIFE CURVE DERIVED FROM TABLE 14

Device Lifetime	Frequency	% or Probability
220	4	.16
230	5	.20
240	5	.20
250	9	.36
260	2	.08
	25	1.00

These results are plotted as probabilities of failure (the life curve) in Fig. 17, and as cumulative probabilities of failures in Fig. 18.

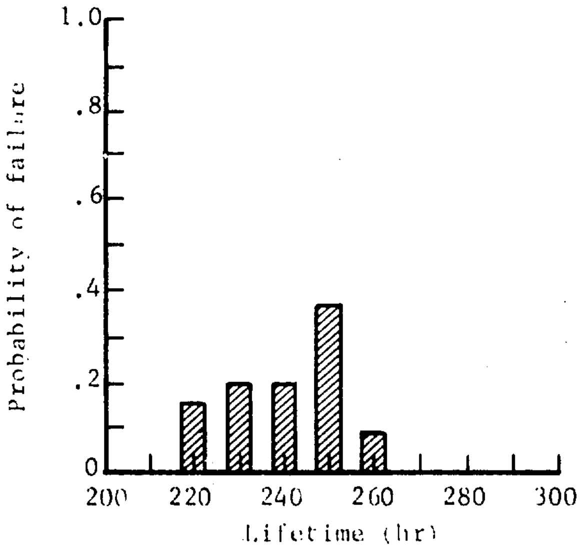

Fig. 17 - Life Curve for Two-Tube Electronic Device

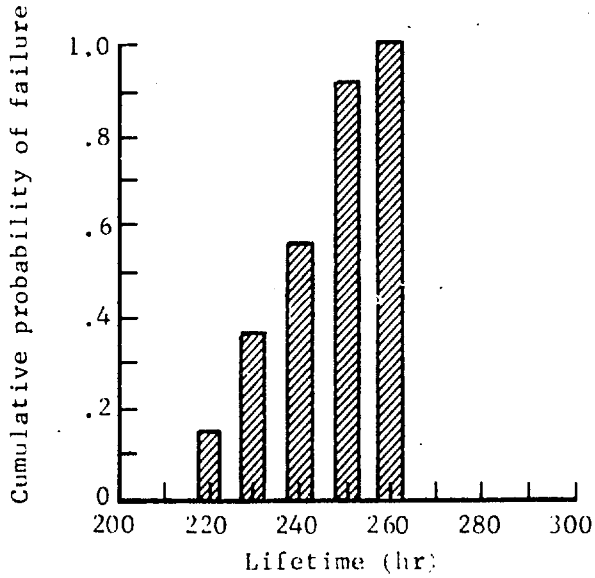

Fig. 18 - Cumulative Probability of Failure for Two-Tube Electronic Device

Appendix D

STATISTICAL MEASURES

Decisionmaking under uncertainty is usually based on a few summary statistics which characterize the probability distribution of the possible results. Those most widely used are measures of central tendency and measures of dispersion. This Appendix reviews the definitions of these measures and also discusses probabilities related to specific levels of performance. It is included for those readers who may wish to briefly review the meaning and definition of these measures.

MEASURES OF CENTRAL TENDENCY

Measures of central tendency indicate where the middle of the distribution lies. The mean is the average, or expected, value. It is given by

$$\mu = E(r) = \sum_i r_i p(r_i) \ . \tag{10}$$

The mode is the most likely value of r; that is, the value that occurs most frequently. In mathematical terms it is that r_j such that $p(r_j) > p(r_i)$ for all $i \neq j$. The median is the value of r that approximately divides the area under the distribution in half.* Mathematically, it is that r_j such that $p(r < r_j) \leq \frac{1}{2}$ and $p(r > r_j) \leq \frac{1}{2}$, where

$$p(r < r_j) = \sum_{i=o}^{i=j-1} p(r_i)$$

and

$$p(r > r_j) = \sum_{i=j+1}^{i=i\ max.} p(r_i) \ .$$

Measures of central tendency are measured in the same units as the variable in question. For example, if r in Table 16 is the speed of

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*If the distribution is continuous, then the median divides the area exactly in half. See Ref. 24, p. 213.

an aircraft in miles per hour, then the mean is μ = 485 miles per hour, the mode is 400 miles per hour, and the median is 500 miles per hour.

MEASURES OF DISPERSION

Measures of dispersion, which indicate the spread about the central tendency, are especially useful in measuring uncertainty. For example, the ultimate outcome from a distribution with high dispersion is more uncertain than the ultimate outcome from a distribution with low dispersion.

Table 16

PROBABILITY DISTRIBUTION FOR
ILLUSTRATION OF STATISTICAL MEASURES

r	p(r)
r_1 = 200	.00
r_2 = 300	.10
r_3 = 400	.35
r_4 = 500	.25
r_5 = 600	.20
r_6 = 700	.10
r_7 = 800	.00

The range, R, is the difference between the values of the variable in question for which the probabilities are not zero. In mathematical terms, the range is given by $r_j - r_i$, where $r_j > r_i$, $p(r_i) > 0$, $p(r_j) > 0$, $p(r_h) = 0$ for all $r_h < r_i$, and $p(r_k) = 0$ for all $r_k > r_j$. The unit of measure of the range is the same as for the variable in question. In Table 16, R = 700 - 300 = 400 miles per hour.

There are limitations to the usefulness of the range. For example, the three distributions shown in Fig. 19 have the same range but they are very different. A more general measure of dispersion that would

distinguish between the three distributions shown in Fig. 19 is the inter-dth percentile range. This is a measure of the central range between which the dth percentage of the distribution lies. In mathematical terms $r_u - r_\ell$ is the dth percentile range when

$$p(r < r_\ell) \le \frac{1-d}{2},\ p(r > r_\ell) \le \frac{1+d}{2},\ \text{and}$$
$$p(r > r_u) \le \frac{1-d}{2},\ p(r < r_u) \le \frac{1+d}{2}\ . \tag{11}$$

As a specific example, if $(1-d)/2 = .25$ and $(1+d)/2 = .75$, then $r_u - r_\ell$ gives the interquartile range. For the example in Table 16, r_ℓ = 400 miles per hour and r_u = 600 miles per hour; hence, the interquartile range is 200 miles per hour.

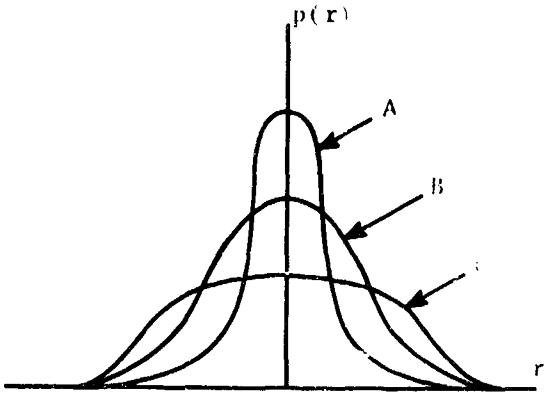

Fig. 19 - Density Functions with the Same Means and Ranges but Different Variances

The variance, σ^2, of a distribution is a measure of the spread, or dispersion, about the mean, μ. For discrete distributions, it can be calculated using the following expression:

$$\sigma^2(r) = \sum_i r_i^2 p(r_i) - \mu^2\ . \tag{12}$$

The unit of measure of the variance is the square of the unit of measure of the variable in question. In Table 16, σ^2 = 13,275 (miles per hour)^2.

If the variance is small, then the area under the distribution is concentrated near the mean. If the variance is large, then the area is widely dispersed. This is illustrated in Fig. 19, where the three density functions A, B, and C have the same mean and the same range but increasing variance. The distributions shown are continuous (not discrete) to simplify the illustration.

The standard deviation, σ, is the positive square root of the variance:

$$\sigma(r) = \sqrt{\sigma^2(r)}. \tag{13}$$

In Table 16, σ = approximately 115 miles per hour.

MEASURES RELATED TO A REQUIRED LEVEL OF PERFORMANCE

In the case of most weapon system development projects, the critical system performance characteristics have specified minimum performance levels usually referred to as specifications. In these cases, the measures of central tendency and dispersion are of limited value because they do not relate directly to the specifications. For example, consider a project to develop an aircraft with a minimum speed (under certain specified conditions) of 750 mph. Assume that the probabilistic information described above has been obtained, and it is found that the expected speed is 700 mph with a standard deviation of 75 mph. Assuming the distribution to be normal, this means that there is a probability of 0.68 that the speed of the aircraft will be between 625 mph and 775 mph. It would be more relevant to have a figure for the probability of meeting or exceeding the required performance level; that is, the probability that the aircraft will fly 750 mph or more. This is easily obtained from the probability distribution over possible speeds. It is obtained by summing the probabilities for all speeds equal to the specified speed and greater. Let r_i indicate the possible speeds of the aircraft and r_s the specified speed. Then

$$p(r \geq r_s) = \sum_{r_i=r_s}^{r_{max}} p(e_i), \tag{14}$$

where r_{max} is the maximum possible speed indicated by the distribution.

The probability of meeting or exceeding the specification is similar to measures of central tendency in that it does not give any indication of how much the specification may be exceeded; that is, no measure of dispersion. Therefore, the probability of meeting or exceeding the specification may be supplemented by a measure such as the probability of exceeding the specifications by more than some percentage, say x. This is given by

$$p(r > r_s + xr_s) = \sum_{r_i=r_s+xr_s}^{r_{max}} p(r_i). \tag{15}$$

This measure is especially useful in situations where a substantial overdesign is undesirable. Continuing the previous example, suppose that the aircraft under development would not be desirable if its speed turned out to be greater than 1000 mph.* This corresponds to exceeding the specifications by 33 percent; that is, $x = 0.33$. If the probability of exceeding 1000 mph is too high,** then some changes in the project may be made.

This measure is also valuable when parallel projects are compared or when progress on one given project is compared with data that has been obtained at two different times (see Sec. III). In such cases, it can be used to discriminate between two situations that may have the

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*The decrease of desirability as speed increases may be due to changes in other performance parameters, such as an increase in the minimum turning radius, or it may reflect a desire to save time and money rather than achieve a higher speed.

**Both x and what probability is "too high" must be determined by the user of the information.

same probabilities of meeting or exceeding the specification but that have different probabilities of exceeding the spectification by x percent.

The difference between the probability of meeting or exceeding the specification and the probability of exceeding the specification by more than x percent is the probability of being within the acceptable range:

$$p(r_s + xr_s > r \geq r_s) = p(r \geq r_s) - p(r > r_s + xr_s). \qquad (16)$$

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10. ABSTRACT

A procedure for measuring the status and progress of a complex system development program. The procedure is based on four premises: (1) Progress is characterized by reduction of uncertainty; hence, if uncertainty can be measured at different times, progress can be indicated by changes in measures of uncertainty. (2) Assessment of uncertainty relative to development is subjective. (3) Subjective estimates of uncertainty can be expressed in terms of probabilities. (4) The amount of uncertainty is indicated by statistical measures of appropriate probability distributions. Consistent with these premises, the procedure for obtaining probability distributions for critical system performance characteristics involves five steps: (1) Obtain design equations. (2) Determine subjective probabilities. (3) Generate probability distributions for system performance. (4) Calculate statistical measures. (5) Compare measures at different times to obtain indications of progress. Methods for determining subjective probabilities should be tested in a real-world development program.

11. KEY WORDS

Research and development
Weapon systems
Probability
Decisionmaking
Uncertainty